Chapter 8 : Other classifications

ENSIIE - Computational complexity theory

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Polynomial approximation

Do we have to return an optimal solution?

Polynomial approximation

Definition

A *polynomial approximation algorithm* for a problem Π is a polynomial algorithm that returns a feasible solution of Π that is "close" to an optimal solution.

Polynomial approximation of a minimization problem

Definition

A *polynomial approximation algorithm* for a minimization problem Π with ratio r or r polynomial approximation algorithm is a polynomial algorithm that, for every instance x of Π, returns a feasible solution of Π such that $M(x, y) \leq r \cdot M^*(x)$.

Polynomial approximation of a maximization problem

Definition

A *polynomial approximation algorithm* for a maximization problem Π with ratio r or r polynomial approximation algorithm is a polynomial algorithm that, for every instance x of Π, returns a feasible solution of Π such that $M(x, y) \geq r \cdot M^*(x)$.

Approximability classes

Approximability classes

- APX: there exists a polynomial approximation algorithm with a constant ratio.
- PTAS: there exists a serie of polynomial approximation algorithms for which the ratio can be infinitely close to 1.
- f-APX: there exists a polynomial approximation algorithm with ratio $f(|x|)$ for every input x, where f is a computable function.

Parameterized complexity

Is the complexity lower when we control some of the parameters of the problem?

Parameter of a problem

Definition

Let Π be a problem, a parameter of Π is an integer k associated to a part of its inputs.

For example

- (CLIQUE PARAM) : Let G be a graph, does G contains a clique of size k?
- (UST PARAM): Let $G = (V, E)$ be a graph and $X = (v_1, v_2, \ldots, v_k) \subset V$, find a tree covering X with minimum size.
- (NPTURING PARAM): Let $\mathcal{M}$ be a Turing machine with x as input, does $\mathcal{M}$ stops in less than k steps?

XP problem

Definition

Let Π be a problem parameterized by k. This problem is XP with respect to k if, when we fix k, Π becomes polynomial. In other words, there exists a computable function f and an algorithm solving Π with complexity $O(n^{f(k)})$.

FPT problem

Definition

Let Π be a problem parameterized by k. This problem is FPT with respect to k if (Fixed Parameter Tractable) there exists a computable function f, a polynomial p and an algorithm solving Π with complexity $O(f(k)p(n))$.

Probabilistic complexity

Should the algorithm be deterministic?

Probabilistic Turing machine

Definition

A *probabilistic Turing machine* is a Turing machine associated with a uniform random bit generator that can be read at any time to take a random decision.

A probabilistic algorithm is an algorithm that take some random decisions.

RP class

Definition

A decision problem Π belongs to the RP class if there exists a probabilistic Turing machine that answers in polynomial time

- NO if the answer is NO;
- YES with probability higher than 1/2 if the answer is YES.

BPP class

Definition

A decision problem Π belongs to the BPP class if there exists a probabilistic Turing machine that answers in polynomial time and that answers correctly with probability higher than 2/3.

ZPP class

Definition

A decision problem Π belongs to the BPP class if there exists a probabilistic Turing machine that answers correctly and that answers in polynomial time in expectation.