Tutorial 2 : Complexity classes

Computational complexity theory, 5th semester.

2022

Exercice 1 — ***Belonging to complexity classes***

For each of the problems in exercice 1 of the previous tutorial, tell to which of the following complexity class that problem belongs : P, NP, Co-NP, PSPACE, EXPTIME, NEXPTIME or EXPSPACE.

Exercice 2 — ***Démonstrations simples***

1. Show that DTIME($f(n)$) $\subset$ NTIME($f(n)$).
2. Show that DTIME($f(n)$) $\subset$ DSPACE($f(n)$).
3. Show that, if NP = P then Co-NP = NP. Is the reciprocate true ?
4. Show that P = Co-P.
5. Using the Savitch theorem, saying that NSPACE($f(n)$) $\subset$ DSPACE($f(n)^2$), prove that PSPACE = NPSPACE and EXPSPACE = NEXPSPACE. Deduce that NPSPACE = Co-NPSPACE and that NEXPSPACE = Co-NEXPSPACE.
6. Using the deterministic time hierarchy theorem saying that DTIME($f(n)$) $\subsetneq$ DTIME($f(n)^2$) (among other results), prove that P $\subsetneq$ EXPTIME.

Exercice 3 — ***PSPACE $\subset$ EXPTIME***

1. Let Π be a PSPACE problem, how much cells of the tape can use a Turing machine that solve Π in polynomial space ?
2. In how many configurations that machine can be ? A configuration is defined by the position of the head, the state pointed by the state register and what is written on the tape.
3. Can the machine be twice in the same configuration during the computation ?
4. Deduce that the complexity of the machine is at most exponential.

Exercice 4 — ***P = NP $\Rightarrow$ EXPTIME = NEXPTIME***

We assume that P = NP.

1. Let Π be a NEXPTIME problem. What is the complexity of a non-deterministic Turing machine solving Π in exponential time ?
2. Let c be a constant, let Π_2 be a decision problem for which the instances are $\{x \cdot \pi^{2^{|x|^c}} \backslash x \subset \{0,1\}^*\}$, where π^d is the symbol π repeated d times. The positive instances are the ones where x encodes a positive instance of Π. Build a non deterministic algorithm that solves Π_2 in polynomial time. To obtain such a complexity, choose well the constant c.
3. Deduce that $\Pi \in$ EXPTIME.
4. Deduce that EXPTIME = NEXPTIME.