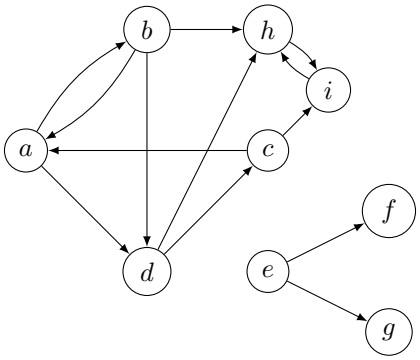
Tutorial 1 : Basic definitions

Graph theory, 1st semester.

2022

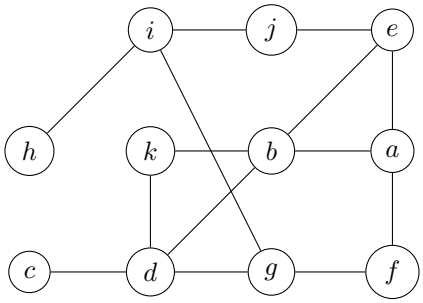
Exercise 1 — *Some definitions*

Let G be the following graph :



1. We write $G = (X, U)$, write X and U.
2. Give $\Gamma(a)$, $\Gamma(b)$ and $\Gamma(c)$.
3. Give the connected components.
4. Give the boolean matrix associated with the connected component containing six nodes.
5. Give an example of path, of cycle, of directed path and of directed cycle of G.
6. Let $G1 = (X1, U1)$ the (induced) subgraph defined by $X1 = \{a, b, c, d\}$. Draw $G1$. Is this graph connected ? Strongly connected ? Answer the same questions with $G2 = (X2, U2)$ and $X2 = \{e, f, g\}$.
7. Give the arcs entering d and its in-degree. Give the arc outgoing from b and its out-degree.
8. Give a partial graph of G and a partial subgraph of G.

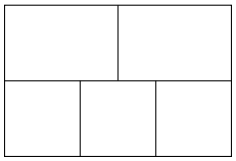
Exercise 2 — *Depth first search and Breadth first search*



1. (a) Run the depth first search algorithm with the previous graph starting from a node of your choice.
   (b) Why can you affirm that this graph is connected ?
   (c) What is the complexity of the algorithm ?

2. (a) Run the breadth first search algorithm with the previous graph starting from a node of your choice.
   (b) Why can you affirm that this graph is connected ?
   (c) What is the complexity of the algorithm ?

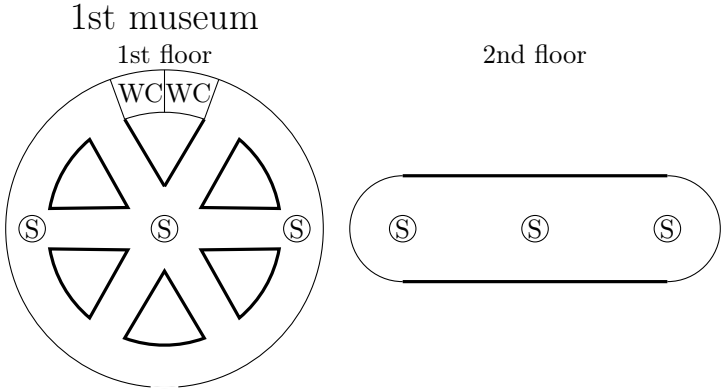
Exercise 3 — *A crossing problem*

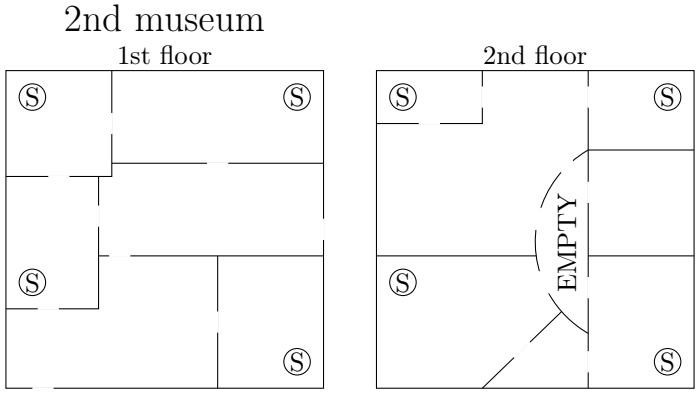


Is it possible to draw a (non-strait) continuous line such that every segment of the previous figure is crossed exactly once by this line ? Model this problem with a graph problem and solve it.

Exercise 4 — *Museum itinerary*

We want to decide the route inside a museum such that no two person goes more than once in front of a painting. The maps of two museum are drawn hereinafter. The rounded S are stairs. In the first museum, paintings are hooked on the walls that are drawn with thick lines. In the second museum, the works are placed every where in the rooms (not only on the wall), except in the empty room.





1. Model this problem in the first museum with a graph problem and solve it.
2. Model this problem in the second museum with a graph problem and solve it.
3. Why is the model not the same ?