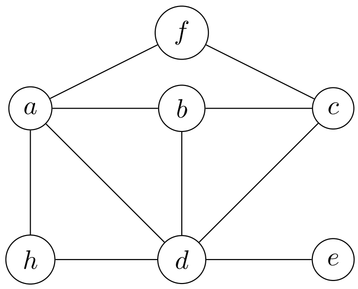
Tutorial 3 :Cliques and independent sets

Graph theory, 1st semester.

2022

Exercise 1 — ***Some definitions***

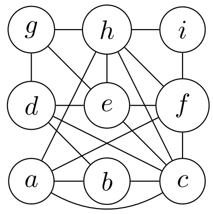
Let G be the following graph :



The weights associated with the nodes are $p(a) = 3$, $p(b) = 2$, $p(c) = 2$, $p(d) = 4$, $p(e) = 1$, $p(f) = 1$ and $p(h) = 1$.

1. Give a maximal independent set of G
2. Give a maximum independent set of G
3. Give a maximum weight independent set of G

Exercise 2 — ***Clique and stable***



1. Find in G a maximal clique, a maximum clique and a partition of the nodes of G into 3 cliques.
2. Show that finding a clique in a graph is equivalent to searching for a maximum independent set in another graph. Which one? Give an example with G.

Exercise 3 — ***Stable set and degree***

Let G be a graph such that $1 \leq d(x_1) \leq \ldots \leq d(x_n)$ and there exists p in $[\![2; n]\!]$ such that $\sum_{i=0}^{p-2} d(x_{n-i}) \leq n - p$. Show that every maximal stable set has at least p nodes.

Exercise 4 — *independent set as a set of equations*

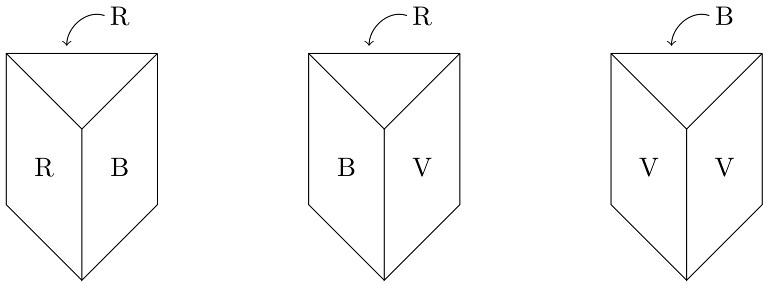
1) Let P be the following problem :

$$(P)\left\{\begin{array}{lll} Max & t+u+v+w+x+y+z & \\ s.c. & & \\ & x+y+z & \leq 1 \\ & z+t+u & \leq 1 \\ & x+v & \leq 1 \\ & t+w & \leq 1 \\ & t,u,v,w,x,y,z \in \{0,1\} & \end{array}\right.$$

1. Show that this problem is equivalent to the search of a maximum independent set in some graph G. Which one ?
2. Find an optimal solution of P.

Exercise 5 — *Stacking*

Let C_1, C_2, C_3 be the three following cylinders (with a triangular base). They are colored on each of their vertical faces (not on the bases). The color of the back face is indicated at the top of each cylinder.



We search for a way to stack every cylinder (they should remain parallel), such that in each column of the stack, each colors appears at most once. Model this problem using a graph problem and solve it.

Exercise 6 — *Wedding placement*

How to place each guest of a wedding such that two people that do not like each other are not at the same table ? Model this problem using a graph problem.