Tutorial 4 : Degrees

Graph theory, 1st semester.

2022

Exercise 1 — *Property of the degree*

Let $G = (V, E)$ be an undirected graph with $n = |V|$ et $m = |E|$.

1. Show that there exists two nodes with the same degree.
2. Show that $\sum_{x \in V} deg(x) = 2m$.
3. Show that the number of odd degree nodes is even.
4. Is that true if the graph is directed ?
5. Is that true if the graph is infinite ?
6. How many edges contains a graph where each node has a degree d ?

Exercise 2 — *My life sucks (on average)* On veut montrer que, dans un réseau social, la plupart des gens ont l'impression d'avoir moins d'amis que leurs amis. On représente le réseau par un graphe non orienté avec n nœuds (les gens) et m arêtes (les liens d'amitiés).

1. Montrer que la moyenne du nombre d'amis d'une personne est $\frac{2m}{n}$
2. On considère la liste contenant, pour chaque personne, pour chaque ami de cette personne, le nombre d'amis de cet ami. Montrer que la moyenne de cette liste, *i.e.* la moyenne du nombre d'amis d'un ami, est $\frac{\sum_{v \in V} deg(v)^2}{2m}$.
3. En déduire qu'une personne moyenne a l'impression d'avoir moins d'ami que ses amis

Exercise 3 — *Hand shakes*

The couple Mrs M and Mr N invite 4 couples. Some people salute each other, some do not. Particularly, the two people of a same couple do not salute each other. Mr N asked everyone how many people they saluted. It appears that each person gave a different answer.

1. Show that
   — the two members of each couple saluted 8 people together.
   — Mrs M and Mr N saluted the same number of people.
2. Is this always true if Mrs M and Mr N are not a couple ?

Exercise 4 — *Simple graphs*

What is the degree of the nodes

— in a clique of size K ?

— in a cycle of size C ?

— in a tree ?

— in a path ?

— in a grid ?

Exercise 5 — *Dense graph and path*

Let G be a graph where the degree of each node is at least d. Show that there exists a chain with $d + 1$ nodes in G.

Exercise 6 — *Highway access*

In a region containing $2p+1$ cities, each city is linked to p other cities by a highway. Show that it is not possible to go from the capital city to any other city using highways.