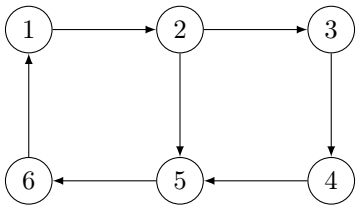
Tutorial 5 : Grundy function, Kernel and Nim game

Graph theory, 1st semester.

2022

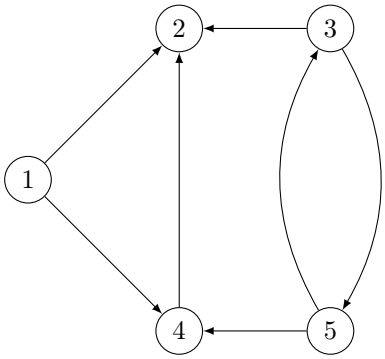
Exercise 1 — *Grundy function*

1. What are the possible Grundy functions of the following graph G ?
2. Deduce two kernels of G.



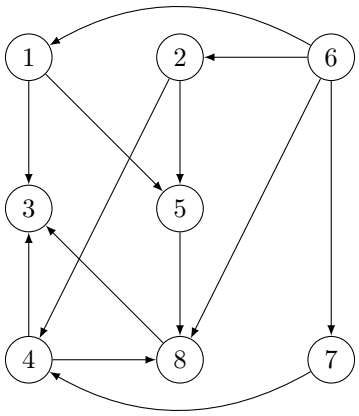
Exercise 2 — *Surveillance*

The following graph G corresponds to a set of points that must be observed. An arc (i, j) means that we can observe j from i. How many guard do we have to put on the graph so that every node is observed ? Model this problem with a graph problem and solve it.



Exercise 3 — *Grundy function in an acyclic graph*

Let $G = (V, A)$ the following graph :



1. Show that, in any acyclic digraph there exists an injective function $f : V \to \mathbb{N}$ such that $(u, v) \in A \Leftrightarrow f(u) < f(v)$.
2. Deduce that G has a unique Grundy function and a kernel.
3. Compute the Grundy function of G and a kernel of G.

Exercise 4 — *Nim games*

We consider the following games. For which game the player 1 or 2 has a winning strategy ? In each game the first player is player 1.

1. Two players have a board with n sticks. At each turn, a player removes 1, 2 or 3 sticks from the board. The player who removes the last sticks looses the game.
2. Two players share a chocolate bar with n lines and m columns of pieces of chocolate. At each turn, a player eats either any number of lines or any number of columns (at least one). The player who finishes the bar looses the game.
3. Two players have a board with a grid where tokens are placed. Some cells of the grid may be empty. At each turn, a player chooses a token, and remove it from the board with all its (horizontal, vertical and diagonal) neighbors. The player who remove the last token wins the game.