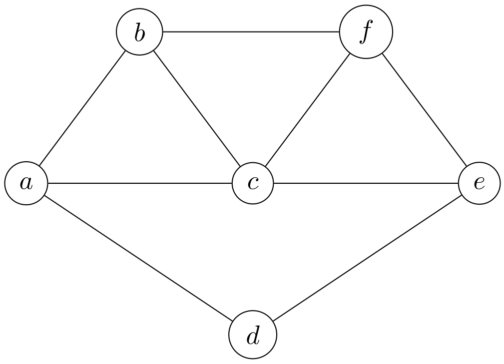
Tutorial 6 : Cycles and cocycles basis

Graph theory, 1st semester.

2022

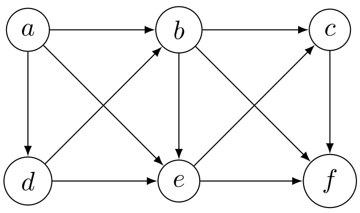
Exercise 1 — ***Search basis***



1. (a) Give the vectors associated with the cycles $(bcfb)$ and $(abfeda)$.
   (b) Show that the cycle $(bcfb)$ can be seen as the linear combination of two other cycles.
   (c) What is the size of a cycles basis of G?
   (d) Give a cycle basis of G that does not contain either $(bcfb)$ or $(abfeda)$.
   (e) Give the linear combination of the vectors of the previous basis that equals $(abfeda)$.
2. (a) Give the vectors associated with (abf) and (ae).
   (b) Show that the cocycle (ae) can be seen as the linear combination of two other cocycles.
   (c) What is the size of a cocycle basis of G?
   (d) Give a cocycle basis of G that does not contain any cocycle with one only node.
   (e) Give the linear combination of the vectors of the previous basis that equals (abf).

Exercise 2 — ***Orthogonalité des cycles et des cocycles***

1. Let G be the following graph, give the cycle vectors $\mu(abcfeda)$ and $\mu(bdefb)$ and the cocycle vectors $\nu(ac)$ and $\nu(bdef)$. Check that each of the two first vectors is orthogonal to the two last.



2. Let $G = (V, E)$ be a graph, v be a node of V and c be a cycle that does not contain v. Show that $\nu(v) \perp \mu(c)$.
3. Let $G = (V, E)$ be a graph, v be a node of V and c be a cycle that contains v. Show that $\nu(v) \perp \mu(c)$.
4. Let $G = (V, E)$ be a graph with p connected components and v_i be a node of the i-th component of G. Show that $B = (\nu(u), u \in V \backslash \{v_1, v_2, \dots, v_p\})$ is a cocycle basis of G.
5. Deduce that, in a graphe G, every cocycle of G is orthogonal to every cycle of G.