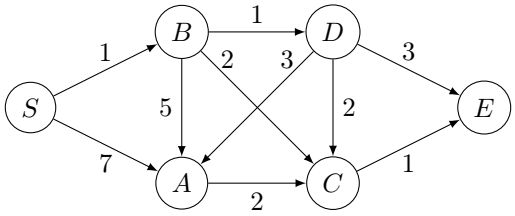
Tutorial 10 : Shortest paths, diameter

Graph theory, 1st semester.

2022

Exercise 1 — *Dijkstra algorithm*

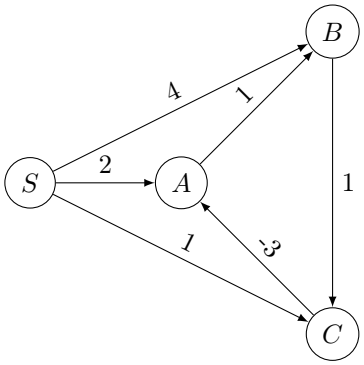
Let G be the following graph :



1. Draw a shortest path tree containing for each node a shortest path from S to that node.
2. In which order the Dijkstra algorithm would enumerate all the nodes of the graph ?
3. Apply the algorithm to compute the weight of the shortest paths from S to every other node.
4. Change the algorithm in order to compute the shortest paths.

Exercise 2 — *Ford-Bellman algorithm*

1. Let G be a weighted graph and s be a node of G. Let $d^k(u)$ be the weight of a shortest path from s to u using at most k arcs. Show that there exists an absorbing directed cycle if and only if there exists a node u such that $d^{n-1}(u) > d^n(u)$.
2. Apply the Bellman-Ford algorithm to the following graph.

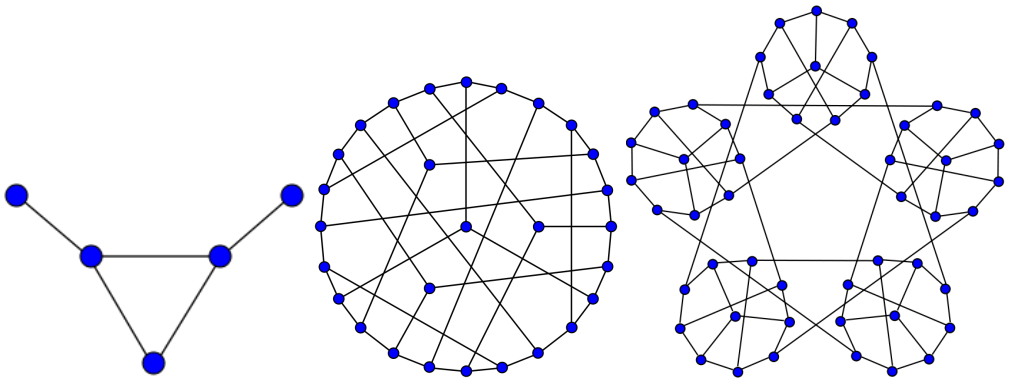


Exercise 3 — *Battery charge*

A small robot is moving on a grid. When it reaches a cell, it can turn left, turn right or go straight right but cannot move back. It cannot fall from the grid. If it does not move it consume 1 joule of energy. If it moves it consume 2 joules. It is also possible that the robot obtain 3 joules when moving because of induction wireless charger placed at some places under the grid. We know where are the chargers. The robot starts with a half empty battery. THis is sufficient to visit al least once every cell of the grid. We want to know if the robot may fully charge its battery by moving.

Model this problem with a graph problem.

Exercise 4 — *Quelques diamètres* What is the diameter of the following graphs ?



(Source : wikipedia, par Koko90)

Exercise 5 — *Diamètre d'un arbre* ...