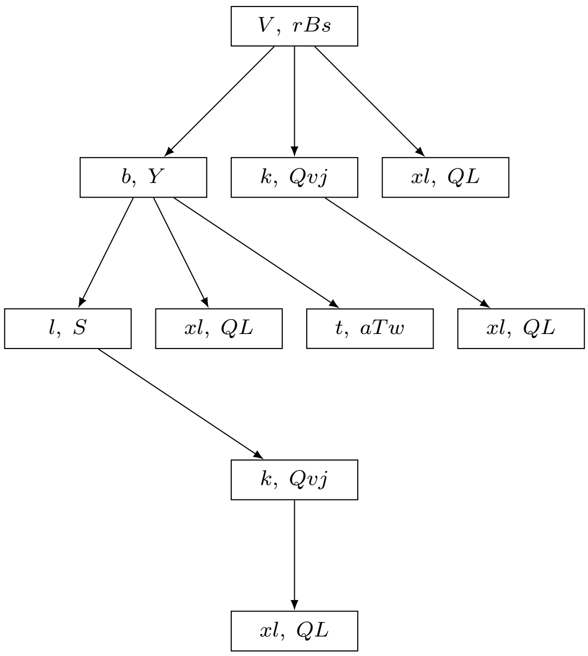
Entraînement - Training

INSTRUCTION : *English version below*
En haut de chaque page se trouvent 3 nombres, par exemple +1/3/58+. Vous ***devez*** *vérifier que, sur chacune des pages de votre sujet, le* ***premier*** *de ces 3 nombres est le même (dans cet exemple, il s'agit donc du 1). Ce nombre identifie votre copie. Les deux autres nombres ne sont pas importants.*
Détacher la dernière feuille et répondre dessus. Ne pas rendre les pages contenant les questions, vous ne devez rendre ***que la dernière feuille****. Chaque question est sur 1 point, aucun point ne sera attribué aux questions contenant une mauvaise réponse.*
Les questions faisant apparaître le symbole ♣ peuvent présenter une ou plusieurs bonnes réponses qui doivent toutes être cochées. Les autres ont une unique bonne réponse.

At the top of each page are written 3 numbers, +1/3/58+. You ***must*** *check that, on each page you have, the* ***first*** *number is the same (in this case, it would be the number 1). This number is the id of your subject. The two other numbers are not important.*
Answer only on the last page. Keep the other pages containing the questions, you just have to return ***the last page****. Each right answer gives you 1 point. For any wrong answer, the mark of the question is 0.*
If there is a question with a symbol ♣, there may be one or more right answer. All of them must be checked. Any other question has only one right answer.

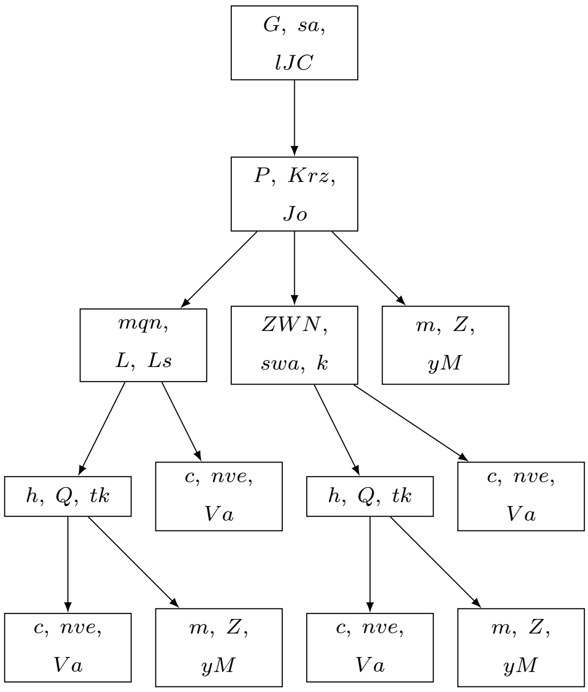
Question 1

Consider a function f with 2 arguments, 2 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(V, rBs)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time ?



1 9	3 3	5 7	7 6	9 5
2 2	4 4	6 10	8 8	10 1

Question 2
Consider a function f with 3 arguments, 3 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(G, sa, lJC)$.
We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[G, sa, lJC], T[P, Krz, Jo], T[ZWN, swa, k], T[h, Q, tk]$ will be computed by f_{ITER}?



1 T[ZWN,swa,k] T[P,Krz,Jo] T[G,sa,lJC] T[h,Q,tk]

2 T[P,Krz,Jo] T[ZWN,swa,k] T[h,Q,tk] T[G,sa,lJC]

3 T[h,Q,tk] T[ZWN,swa,k] T[P,Krz,Jo] T[G,sa,lJC]

4 T[P,Krz,Jo] T[h,Q,tk] T[ZWN,swa,k] T[G,sa,lJC]

5 T[h,Q,tk] T[P,Krz,Jo] T[ZWN,swa,k] T[G,sa,lJC]

6 T[P,Krz,Jo] T[G,sa,lJC] T[h,Q,tk] T[ZWN,swa,k]

7 T[G,sa,lJC] T[ZWN,swa,k] T[P,Krz,Jo] T[h,Q,tk]

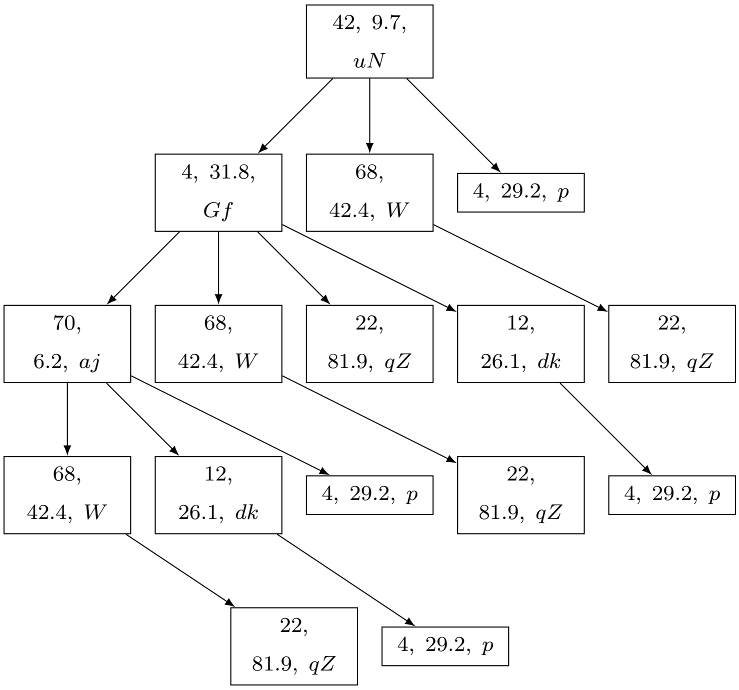
8 T[G,sa,lJC] T[P,Krz,Jo] T[h,Q,tk] T[ZWN,swa,k]

9 T[h,Q,tk] T[ZWN,swa,k] T[G,sa,lJC] T[P,Krz,Jo]

Question 3
We consider a function f with 4 arguments, that are respectively in the sets F, C, F and C. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in F \times C \times F \times C$, in order to compute $f(x)$, we have to execute at most $O(|C|^2 \cdot |F|)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(1)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

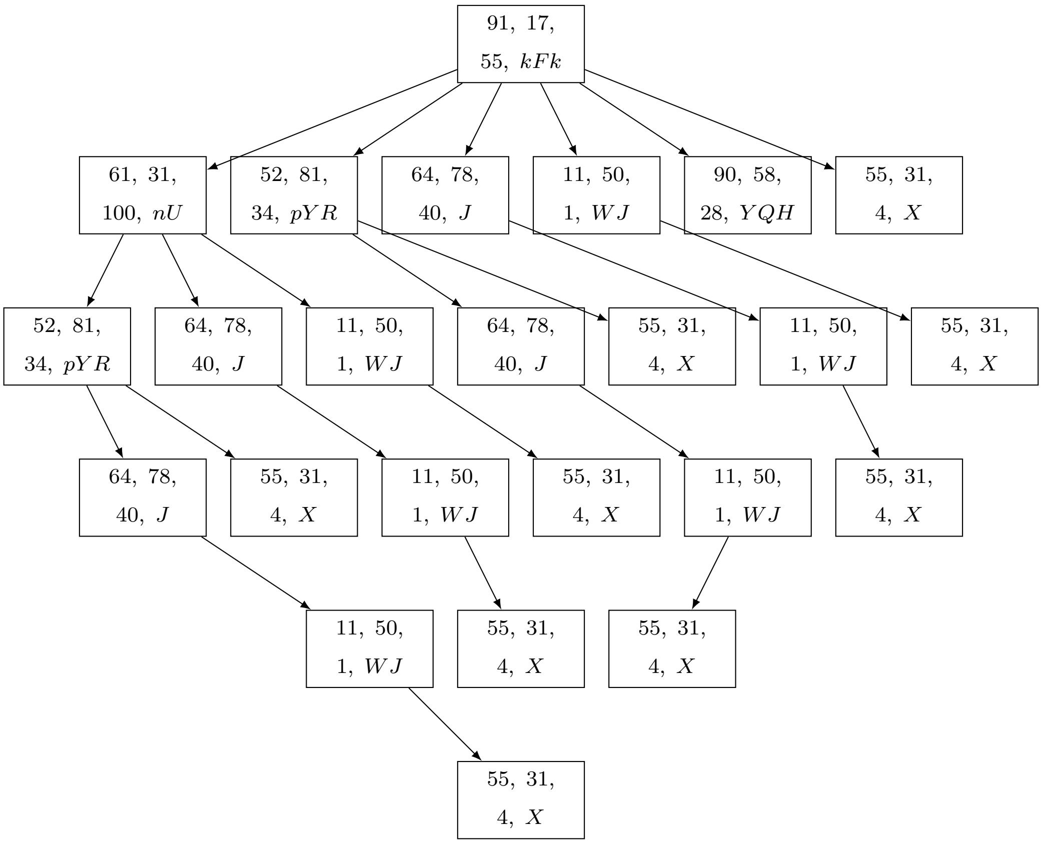
- [1] $O(|C|^3 \cdot |F|^6)$
- [2] $O(|C|^8 \cdot |F|)$
- [3] $O(|C|^6 \cdot |F|^2)$
- [4] $O(|C|^4 \cdot |F|^2)$
- [5] $O(|C|^5 \cdot |F|^4)$
- [6] $O(|C|^7 \cdot |F|^5)$
- [7] $O(|C|^2)$
- [8] $O(|C|^4 \cdot |F|^3)$
- [9] $O(|C|^5 \cdot |F|^6)$
- [10] $O(|F|)$

Question 4
Consider a function f with 3 arguments, 1 integer, 1 real, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(42, 9.7, uN)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time ?



1 6	3 8	5 11	7 5	9 15
2 14	4 3	6 12	8 2	10 13

Question 5
Consider a function f with 4 arguments, 3 integers, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(91, 17, 55, kFk)$.
We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[52, 81, 34, pYR], T[55, 31, 4, X], T[90, 58, 28, YQH], T[91, 17, 55, kFk]$ will be computed by f_{ITER}?



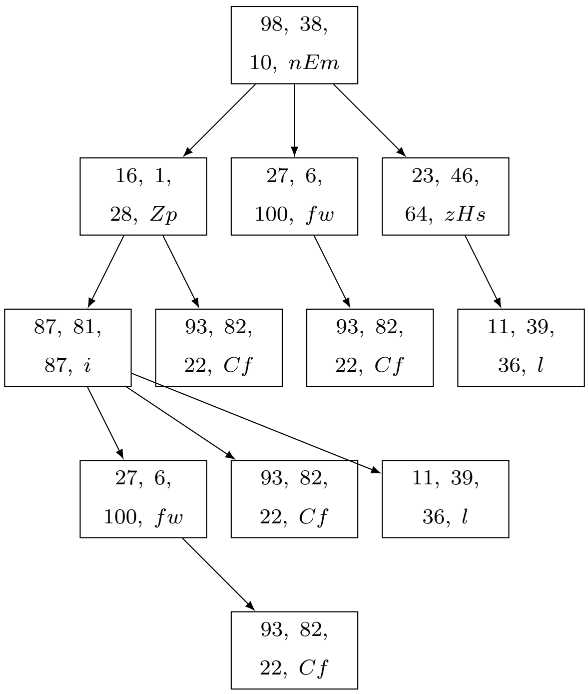
- [ ] 1 T[55,31,4,X] T[90,58,28,YQH] T[91,17,55,kFk] T[52,81,34,pYR]
- [ ] 2 T[55,31,4,X] T[91,17,55,kFk] T[90,58,28,YQH] T[52,81,34,pYR]
- [ ] 3 T[55,31,4,X] T[91,17,55,kFk] T[52,81,34,pYR] T[90,58,28,YQH]
- [ ] 4 T[52,81,34,pYR] T[55,31,4,X] T[90,58,28,YQH] T[91,17,55,kFk]
- [ ] 5 T[55,31,4,X] T[90,58,28,YQH] T[52,81,34,pYR] T[91,17,55,kFk]
- [ ] 6 T[91,17,55,kFk] T[90,58,28,YQH] T[55,31,4,X] T[52,81,34,pYR]
- [ ] 7 T[55,31,4,X] T[52,81,34,pYR] T[91,17,55,kFk] T[90,58,28,YQH]
- [ ] 8 T[90,58,28,YQH] T[52,81,34,pYR] T[55,31,4,X] T[91,17,55,kFk]
- [ ] 9 T[91,17,55,kFk] T[90,58,28,YQH] T[52,81,34,pYR] T[55,31,4,X]

Question 6

We consider a function f with 4 arguments, that are respectively in the sets E, D, A and E. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in E \times D \times A \times E$, in order to compute $f(x)$, we have to execute at most $O(|A|^2 \cdot |D|^3 \cdot |E|)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|D| \cdot |E|)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

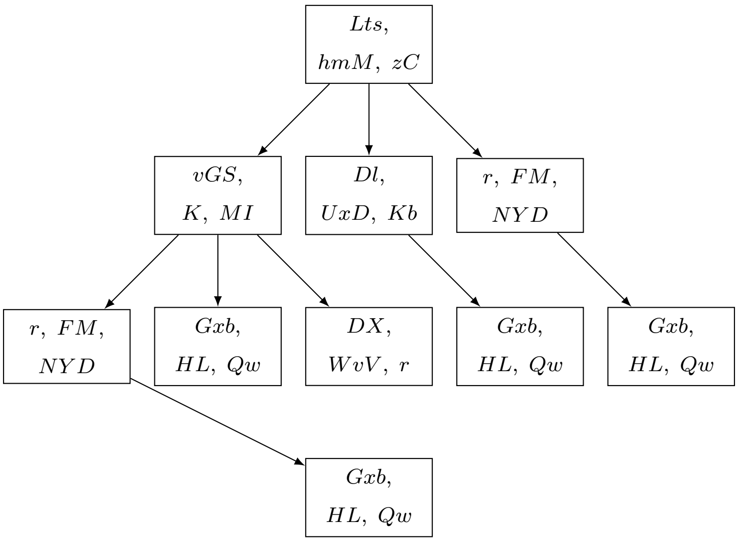
- 1 $O(|A| \cdot |D| \cdot |E|^6)$
- 2 $O(|A|^6 \cdot |D|^3 \cdot |E|^2)$
- 3 $O(|A|^2 \cdot |D|^3 \cdot |E|^4)$
- 4 $O(|A|^3 \cdot |D|^4 \cdot |E|^3)$
- 5 $O(|D|^6 \cdot |E|^2)$
- 6 $O(|D| \cdot |E|^6)$
- 7 $O(|A|^4 \cdot |D|^3 \cdot |E|^2)$
- 8 $O(|D|^7 \cdot |E|^4)$
- 9 $O(|A|^5 \cdot |D|^6)$
- 10 $O(|A|^6 \cdot |D|^6 \cdot |E|^5)$

Question 7
Consider a function f with 4 arguments, 3 integers, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(98, 38, 10, nEm)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time ?



1 6	3 5	5 8	7 12	9 7
2 2	4 11	6 9	8 1	10 3

Question 8
Consider a function f with 3 arguments, 3 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(Lts, hmM, zC)$. We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[DX, WvV, r], T[Gxb, HL, Qw], T[Lts, hmM, zC], T[r, FM, NYD], T[vGS, K, MI]$ will be computed by f_{ITER}?



1. T[Gxb,HL,Qw] T[DX,WvV,r] T[r,FM,NYD] T[Lts,hmM,zC] T[vGS,K,MI]
2. T[Gxb,HL,Qw] T[Lts,hmM,zC] T[vGS,K,MI] T[DX,WvV,r] T[r,FM,NYD]
3. T[Lts,hmM,zC] T[vGS,K,MI] T[Gxb,HL,Qw] T[r,FM,NYD] T[DX,WvV,r]
4. T[r,FM,NYD] T[DX,WvV,r] T[Lts,hmM,zC] T[Gxb,HL,Qw] T[vGS,K,MI]
5. T[DX,WvV,r] T[Gxb,HL,Qw] T[r,FM,NYD] T[vGS,K,MI] T[Lts,hmM,zC]
6. T[Gxb,HL,Qw] T[vGS,K,MI] T[DX,WvV,r] T[Lts,hmM,zC] T[r,FM,NYD]
7. T[DX,WvV,r] T[r,FM,NYD] T[Gxb,HL,Qw] T[vGS,K,MI] T[Lts,hmM,zC]
8. T[Gxb,HL,Qw] T[Lts,hmM,zC] T[DX,WvV,r] T[vGS,K,MI] T[r,FM,NYD]
9. T[Lts,hmM,zC] T[vGS,K,MI] T[Gxb,HL,Qw] T[DX,WvV,r] T[r,FM,NYD]

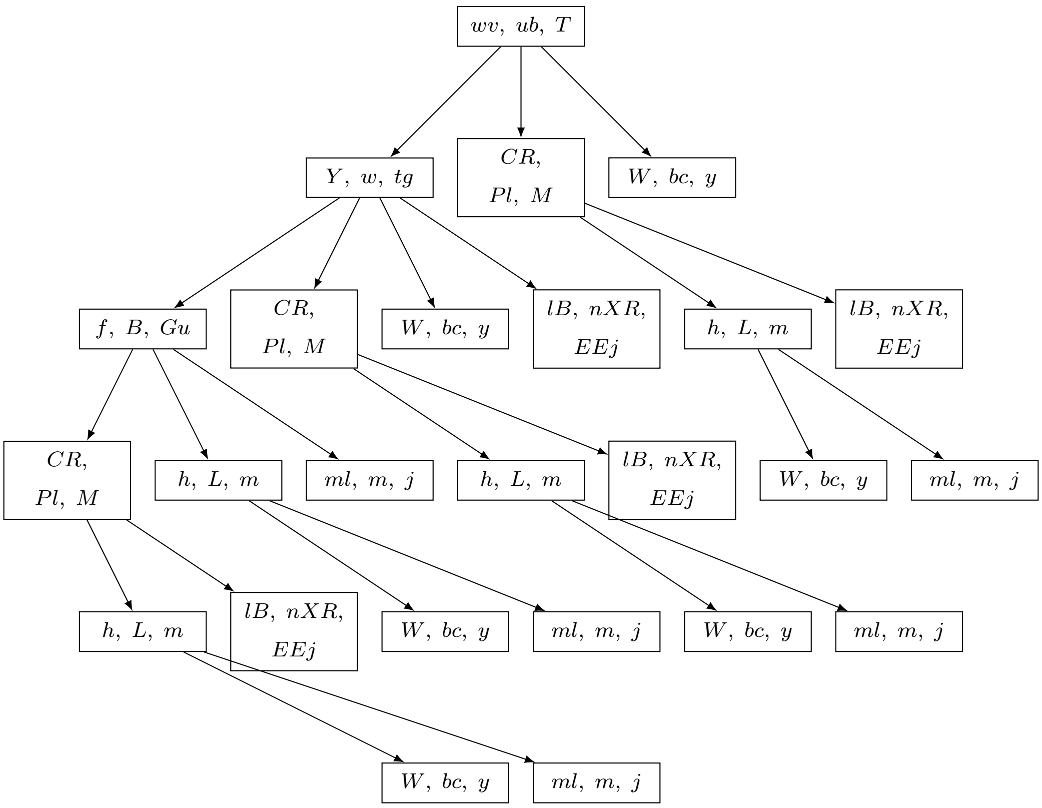
Question 9

We consider a function f with 3 arguments, that are respectively in the sets C, A and E. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in C \times A \times E$, in order to compute $f(x)$, we have to execute at most $O(|A|^3 \cdot |C|^3)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|A|^3 \cdot |E|^2)$.

Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

- [1] $O(|A|^3 \cdot |C|^4 \cdot |E|^6)$
- [2] $O(|A|^5 \cdot |C|^2)$
- [3] $O(|A|^2 \cdot |C|^7 \cdot |E|^2)$
- [4] $O(|A|^3 \cdot |C|^3)$
- [5] $O(|A|^4 \cdot |C|^4 \cdot |E|^3)$
- [6] $O(|A|^4 \cdot |C|^7 \cdot |E|^2)$
- [7] $O(|A|^5 \cdot |C|^5 \cdot |E|^6)$
- [8] $O(|A|^6 \cdot |C|^5 \cdot |E|^4)$
- [9] $O(|A|^6 \cdot |C|^2 \cdot |E|^4)$
- [10] $O(|A|^5 \cdot |C|^8 \cdot |E|^6)$

Question 10
Consider a function f with 3 arguments, 3 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(wv, ub, T)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time?

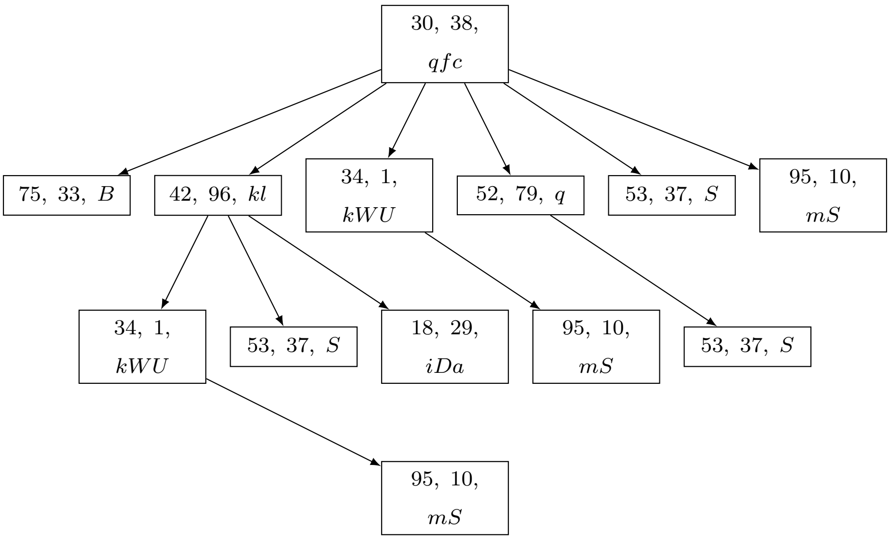


1 7	3 11	5 13	7 5	9 10
2 8	4 4	6 2	8 3	10 14

Question 11

Consider a function f with 3 arguments, 2 integers, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(30, 38, qfc)$.

We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[18, 29, iDa], T[30, 38, qfc], T[42, 96, kl], T[75, 33, B]$ will be computed by f_{ITER}?



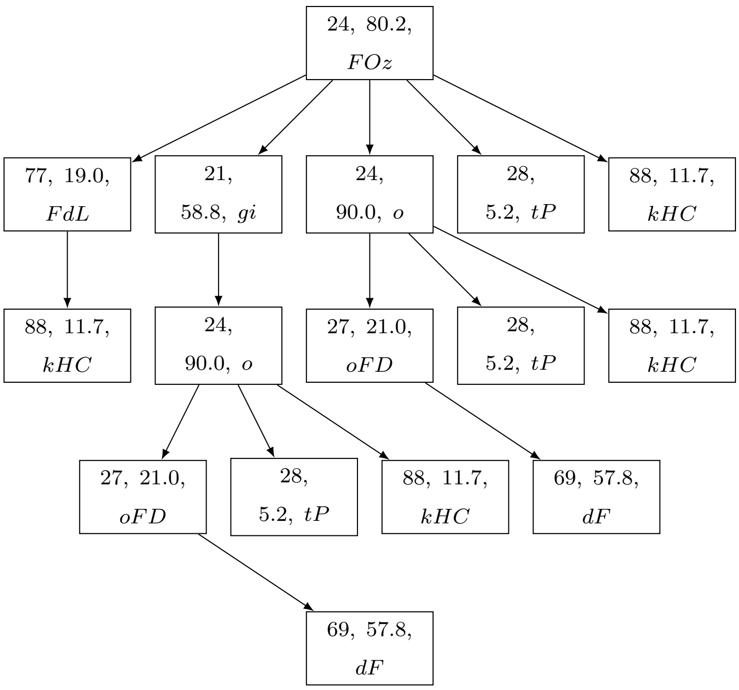
1. T[42,96,kl] T[75,33,B] T[30,38,qfc] T[18,29,iDa]
2. T[30,38,qfc] T[18,29,iDa] T[42,96,kl] T[75,33,B]
3. T[42,96,kl] T[18,29,iDa] T[30,38,qfc] T[75,33,B]
4. T[75,33,B] T[30,38,qfc] T[18,29,iDa] T[42,96,kl]
5. T[18,29,iDa] T[42,96,kl] T[75,33,B] T[30,38,qfc]
6. T[75,33,B] T[30,38,qfc] T[42,96,kl] T[18,29,iDa]
7. T[42,96,kl] T[75,33,B] T[18,29,iDa] T[30,38,qfc]
8. T[18,29,iDa] T[30,38,qfc] T[42,96,kl] T[75,33,B]
9. T[42,96,kl] T[18,29,iDa] T[75,33,B] T[30,38,qfc]

Question 12

We consider a function f with 2 arguments, that are respectively in the sets E and A. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in E \times A$, in order to compute $f(x)$, we have to execute at most $O(|A|^3 \cdot |E|^2)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|E|^3)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

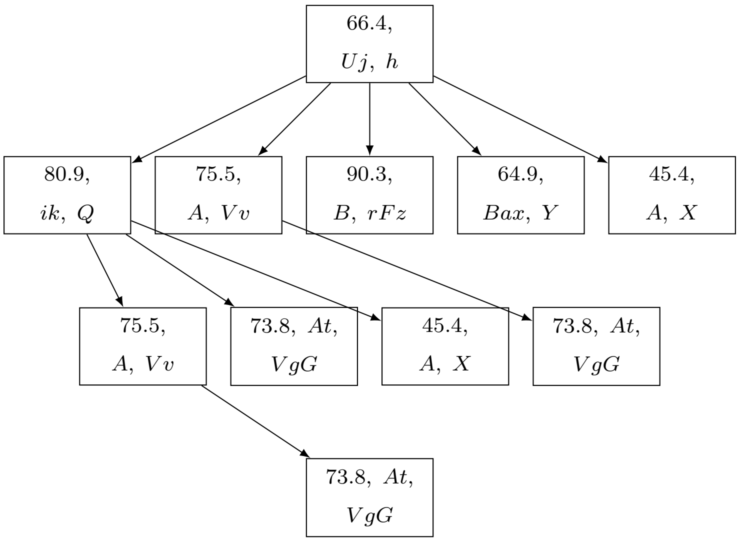
- 1 $O(|A|^4 \cdot |E|^4)$
- 2 $O(|A|^5 \cdot |E|^4)$
- 3 $O(|A|^7 \cdot |E|^4)$
- 4 $O(|A|^7 \cdot |E|^6)$
- 5 $O(|A|^3 \cdot |E|^7)$
- 6 $O(|A| \cdot |E|^7)$
- 7 $O(|A|^2)$
- 8 $O(|A|^6 \cdot |E|^2)$
- 9 $O(|A|^7 \cdot |E|^7)$
- 10 $O(|A| \cdot |E|^5)$

Question 13
Consider a function f with 3 arguments, 1 integer, 1 real, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(24, 80.2, FOz)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time ?



1 7	3 9	5 8	7 10	9 15
2 2	4 14	6 13	8 11	10 5

Question 14
Consider a function f with 3 arguments, 1 real, 2 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(66.4, Uj, h)$. We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[45.4, A, X], T[64.9, Bax, Y], T[75.5, A, Vv], T[80.9, ik, Q]$ will be computed by f_{ITER}?

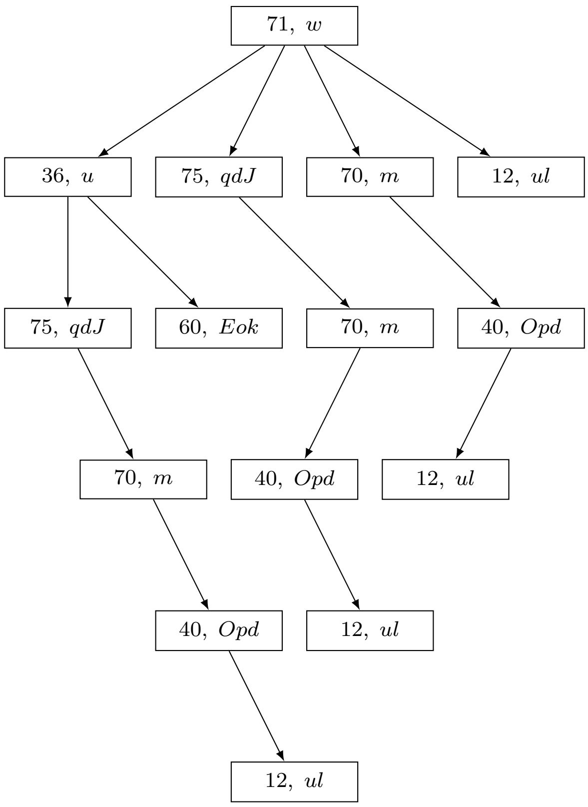


1. T[45.4,A,X]
   T[80.9,ik,Q]
   T[75.5,A,Vv]
   T[64.9,Bax,Y]
2. T[80.9,ik,Q]
   T[75.5,A,Vv]
   T[64.9,Bax,Y]
   T[45.4,A,X]
3. T[45.4,A,X]
   T[64.9,Bax,Y]
   T[75.5,A,Vv]
   T[80.9,ik,Q]
4. T[75.5,A,Vv]
   T[80.9,ik,Q]
   T[64.9,Bax,Y]
   T[45.4,A,X]
5. T[45.4,A,X]
   T[64.9,Bax,Y]
   T[80.9,ik,Q]
   T[75.5,A,Vv]
6. T[80.9,ik,Q]
   T[75.5,A,Vv]
   T[45.4,A,X]
   T[64.9,Bax,Y]
7. T[64.9,Bax,Y]
   T[80.9,ik,Q]
   T[45.4,A,X]
   T[75.5,A,Vv]
8. T[80.9,ik,Q]
   T[64.9,Bax,Y]
   T[75.5,A,Vv]
   T[45.4,A,X]
9. T[80.9,ik,Q]
   T[45.4,A,X]
   T[64.9,Bax,Y]
   T[75.5,A,Vv]

Question 15
We consider a function f with 2 arguments, that are respectively in the sets D and C. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in D \times C$, in order to compute $f(x)$, we have to execute at most $O(|C|^2)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|C|^3 \cdot |D|)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

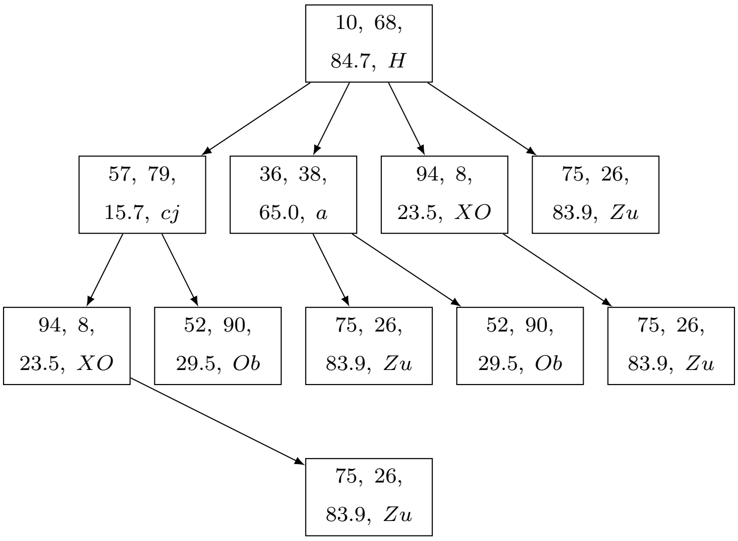
- [1] $O(|C|^4)$
- [2] $O(|C|^5 \cdot |D|)$
- [3] $O(|C|^7 \cdot |D|^2)$
- [4] $O(|C|^2 \cdot |D|^3)$
- [5] $O(|C|^2 \cdot |D|^4)$
- [6] $O(|C|^7 \cdot |D|^4)$
- [7] $O(|C|^3 \cdot |D|)$
- [8] $O(|C|^4 \cdot |D|^2)$
- [9] $O(|C|^3 \cdot |D|^4)$
- [10] $O(|C|^6)$

Question 16
Consider a function f with 2 arguments, 1 integer, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(71, w)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time ?



1 2	3 15	5 1	7 5	9 6
2 12	4 3	6 4	8 13	10 11

Question 17
Consider a function f with 4 arguments, 2 integers, 1 real, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(10, 68, 84.7, H)$.
We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[10, 68, 84.7, H], T[36, 38, 65.0, a], T[52, 90, 29.5, Ob], T[57, 79, 15.7, cj], T[94, 8, 23.5, XO]$ will be computed by f_{ITER}?



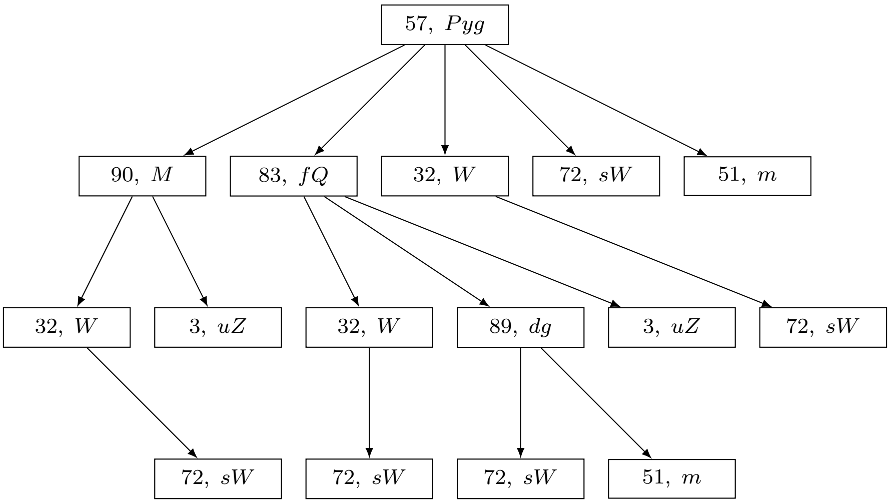
- [1] T[10,68,84.7,H] T[94,8,23.5,XO] T[52,90,29.5,Ob] T[36,38,65.0,a] T[57,79,15.7,cj]
- [2] T[94,8,23.5,XO] T[10,68,84.7,H] T[36,38,65.0,a] T[52,90,29.5,Ob] T[57,79,15.7,cj]
- [3] T[10,68,84.7,H] T[36,38,65.0,a] T[94,8,23.5,XO] T[57,79,15.7,cj] T[52,90,29.5,Ob]
- [4] T[52,90,29.5,Ob] T[94,8,23.5,XO] T[36,38,65.0,a] T[57,79,15.7,cj] T[10,68,84.7,H]
- [5] T[57,79,15.7,cj] T[94,8,23.5,XO] T[36,38,65.0,a] T[10,68,84.7,H] T[52,90,29.5,Ob]
- [6] T[57,79,15.7,cj] T[94,8,23.5,XO] T[52,90,29.5,Ob] T[10,68,84.7,H] T[36,38,65.0,a]
- [7] T[36,38,65.0,a] T[52,90,29.5,Ob] T[94,8,23.5,XO] T[10,68,84.7,H] T[57,79,15.7,cj]
- [8] T[57,79,15.7,cj] T[10,68,84.7,H] T[36,38,65.0,a] T[94,8,23.5,XO] T[52,90,29.5,Ob]
- [9] T[57,79,15.7,cj] T[10,68,84.7,H] T[36,38,65.0,a] T[52,90,29.5,Ob] T[94,8,23.5,XO]

Question 18

We consider a function f with 3 arguments, that are respectively in the sets C, A and E. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in C \times A \times E$, in order to compute $f(x)$, we have to execute at most $O(|A|^3 \cdot |C|^2)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|A| \cdot |C|^3 \cdot |E|^2)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

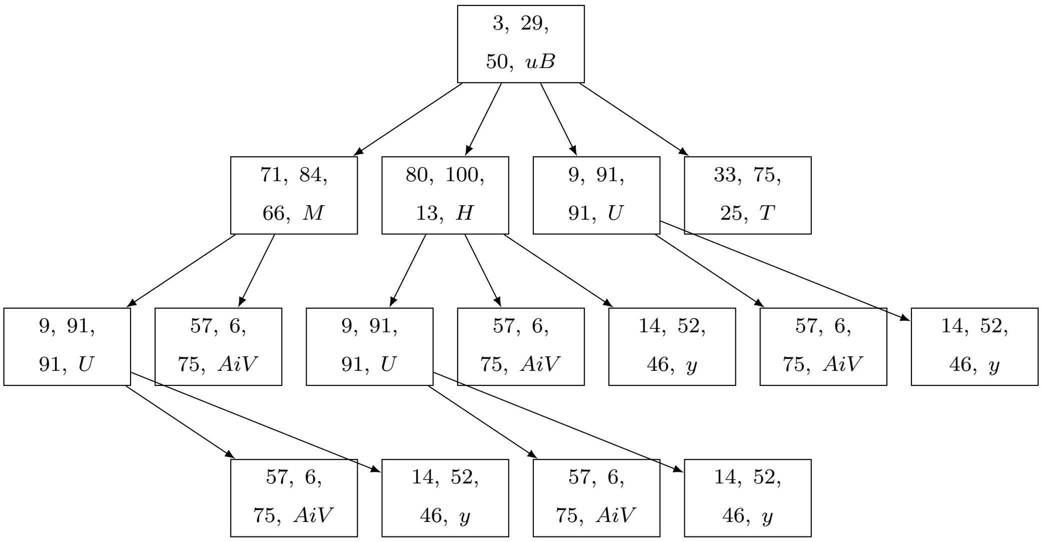
- [1] $O(|A|^5 \cdot |E|^2)$
- [2] $O(|A|^4 \cdot |C|^4 \cdot |E|^3)$
- [3] $O(|A|^8 \cdot |E|^3)$
- [4] $O(|A|^7 \cdot |C|^8 \cdot |E|^6)$
- [5] $O(|A|^8 \cdot |E|^6)$
- [6] $O(|A|^2 \cdot |C|^6 \cdot |E|^6)$
- [7] $O(|A|^5 \cdot |C|^8)$
- [8] $O(|A|^2 \cdot |C| \cdot |E|^6)$
- [9] $O(|A|^5 \cdot |C| \cdot |E|^4)$
- [10] $O(|C|^5 \cdot |E|^4)$

Question 19
Consider a function f with 2 arguments, 1 integer, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(57, Pyg)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time ?



1 13	3 6	5 7	7 2	9 5
2 10	4 3	6 14	8 15	10 9

Question 20
Consider a function f with 4 arguments, 3 integers, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(3, 29, 50, uB)$.
We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[3, 29, 50, uB], T[57, 6, 75, AiV], T[80, 100, 13, H], T[9, 91, 91, U]$ will be computed by f_{ITER}?



☐ 1 T[3,29,50,uB] T[9,91,91,U] T[80,100,13,H] T[57,6,75,AiV]

☐ 2 T[57,6,75,AiV] T[3,29,50,uB] T[80,100,13,H] T[9,91,91,U]

☐ 3 T[3,29,50,uB] T[9,91,91,U] T[57,6,75,AiV] T[80,100,13,H]

☐ 4 T[80,100,13,H] T[3,29,50,uB] T[57,6,75,AiV] T[9,91,91,U]

☐ 5 T[57,6,75,AiV] T[9,91,91,U] T[3,29,50,uB] T[80,100,13,H]

☐ 6 T[80,100,13,H] T[9,91,91,U] T[57,6,75,AiV] T[3,29,50,uB]

☐ 7 T[57,6,75,AiV] T[9,91,91,U] T[80,100,13,H] T[3,29,50,uB]

☐ 8 T[3,29,50,uB] T[57,6,75,AiV] T[80,100,13,H] T[9,91,91,U]

☐ 9 T[80,100,13,H] T[57,6,75,AiV] T[9,91,91,U] T[3,29,50,uB]

Question 21

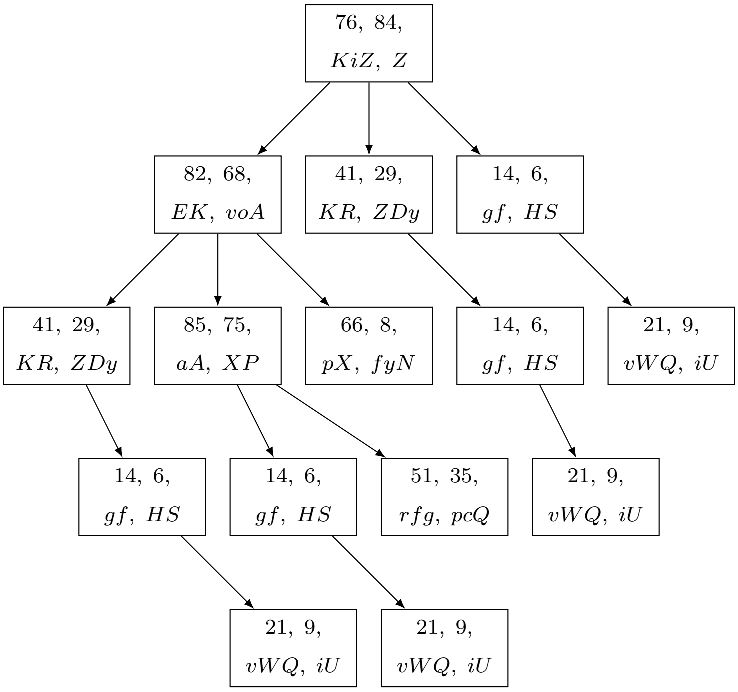
We consider a function f with 3 arguments, that are respectively in the sets A, C and C. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in A \times C \times C$, in order to compute $f(x)$, we have to execute at most $O(|A|^3)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|A|^2 \cdot |C|^3)$.

Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

- [1] $O(|A|^8 \cdot |C|^4)$
- [2] $O(|A|^5 \cdot |C|^7)$
- [3] $O(|A|^3 \cdot |C|^3)$
- [4] $O(|A|)$
- [5] $O(|A|^2 \cdot |C|^8)$
- [6] $O(|A|^2 \cdot |C|^4)$
- [7] $O(|A|^3 \cdot |C|^7)$
- [8] $O(|A|^4 \cdot |C|^9)$
- [9] $O(|A|^4 \cdot |C|^5)$
- [10] $O(|A|^4 \cdot |C|^2)$

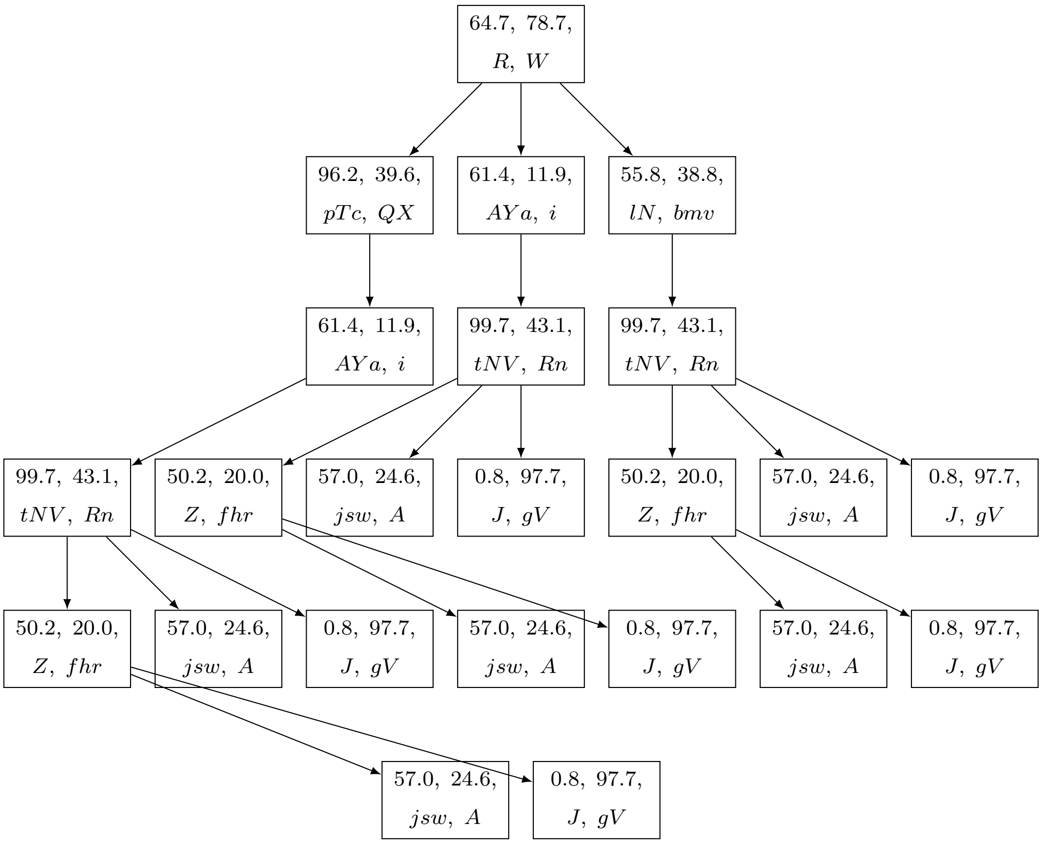
Question 22

Consider a function f with 4 arguments, 2 integers, 2 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(76, 84, KiZ, Z)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time?



- [1] 5
- [2] 15
- [3] 9
- [4] 13
- [5] 8
- [6] 10
- [7] 1
- [8] 6
- [9] 7
- [10] 12

Question 23
Consider a function f with 4 arguments, 2 reals, 2 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(64.7, 78.7, R, W)$.
We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[55.8, 38.8, lN, bmv], T[57.0, 24.6, jsw, A], T[64.7, 78.7, R, W], T[99.7, 43.1, tNV, Rn]$ will be computed by f_{ITER}?



1 T[64.7,78.7,R,W]
T[55.8,38.8,lN,bmv]
T[57.0,24.6,jsw,A]
T[99.7,43.1,tNV,Rn]

2 T[99.7,43.1,tNV,Rn]
T[57.0,24.6,jsw,A]
T[64.7,78.7,R,W]
T[55.8,38.8,lN,bmv]

3 T[57.0,24.6,jsw,A]
T[55.8,38.8,lN,bmv]
T[64.7,78.7,R,W]
T[99.7,43.1,tNV,Rn]

4 T[57.0,24.6,jsw,A]
T[99.7,43.1,tNV,Rn]
T[64.7,78.7,R,W]
T[55.8,38.8,lN,bmv]

5 T[57.0,24.6,jsw,A]
T[55.8,38.8,lN,bmv]
T[99.7,43.1,tNV,Rn]
T[64.7,78.7,R,W]

6 T[99.7,43.1,tNV,Rn]
T[55.8,38.8,lN,bmv]
T[57.0,24.6,jsw,A]
T[64.7,78.7,R,W]

7 T[55.8,38.8,lN,bmv]
T[64.7,78.7,R,W]
T[99.7,43.1,tNV,Rn]
T[57.0,24.6,jsw,A]

8 T[57.0,24.6,jsw,A]
T[99.7,43.1,tNV,Rn]
T[55.8,38.8,lN,bmv]
T[64.7,78.7,R,W]

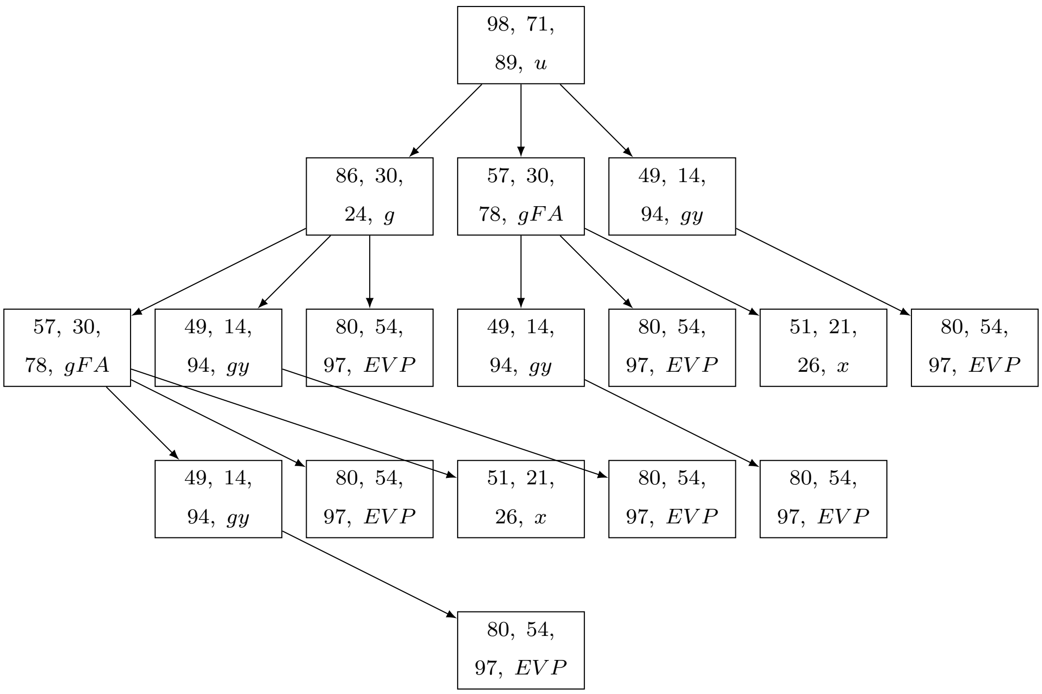
9 T[99.7,43.1,tNV,Rn]
T[57.0,24.6,jsw,A]
T[55.8,38.8,lN,bmv]
T[64.7,78.7,R,W]

Question 24

We consider a function f with 3 arguments, that are respectively in the sets A, F and D. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in A \times F \times D$, in order to compute $f(x)$, we have to execute at most $O(|A|^2 \cdot |D|^3)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|A| \cdot |D|^3 \cdot |F|^2)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

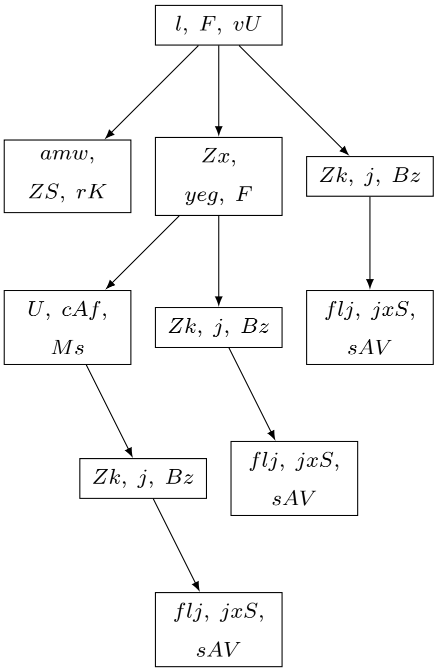
- 1 $O(|D|^3 \cdot |F|)$
- 2 $O(|A|^2 \cdot |D|^2 \cdot |F|^3)$
- 3 $O(|A| \cdot |D|^5 \cdot |F|^4)$
- 4 $O(|A|^3 \cdot |D|^3 \cdot |F|)$
- 5 $O(|A| \cdot |D|^7 \cdot |F|^5)$
- 6 $O(|A|^3 \cdot |D|^5 \cdot |F|^4)$
- 7 $O(|A|^5 \cdot |D|^3 \cdot |F|^5)$
- 8 $O(|A|^3 \cdot |D|^4 \cdot |F|^3)$
- 9 $O(|A|^3 \cdot |D|^8 \cdot |F|^5)$
- 10 $O(|D|^2 \cdot |F|^5)$

Question 25
Consider a function f with 4 arguments, 3 integers, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(98, 71, 89, u)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time ?



- [1] 3
- [2] 1
- [3] 4
- [4] 6
- [5] 7
- [6] 11
- [7] 12
- [8] 5
- [9] 13
- [10] 8

Question 26
Consider a function f with 3 arguments, 3 strings. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(l, F, vU)$.
We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[U, cAf, Ms], T[Zk, j, Bz], T[amw, ZS, rK], T[flj, jxS, sAV], T[l, F, vU]$ will be computed by f_{ITER}?

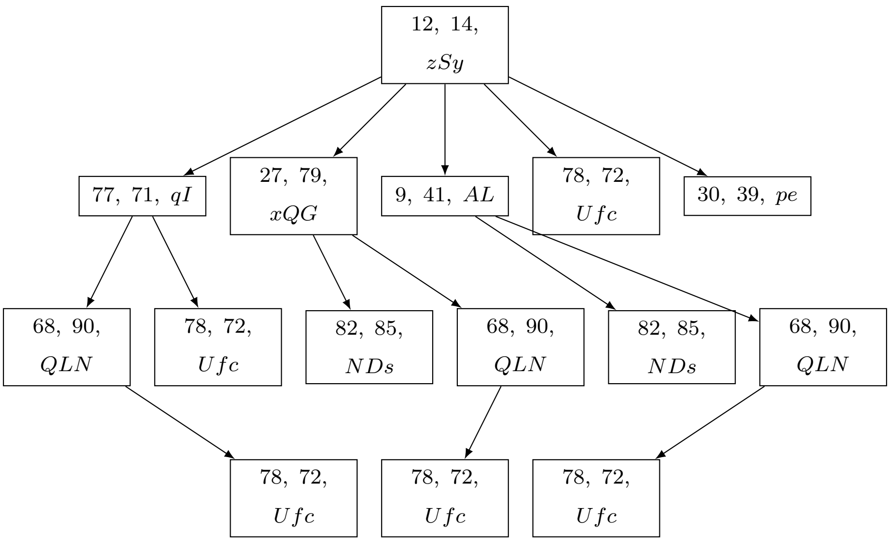


1. T[Zk,j,Bz] T[l,F,vU] T[amw,ZS,rK] T[flj,jxS,sAV] T[U,cAf,Ms]
2. T[amw,ZS,rK] T[l,F,vU] T[U,cAf,Ms] T[Zk,j,Bz] T[flj,jxS,sAV]
3. T[U,cAf,Ms] T[flj,jxS,sAV] T[amw,ZS,rK] T[l,F,vU] T[Zk,j,Bz]
4. T[flj,jxS,sAV] T[Zk,j,Bz] T[amw,ZS,rK] T[l,F,vU] T[U,cAf,Ms]
5. T[amw,ZS,rK] T[l,F,vU] T[flj,jxS,sAV] T[U,cAf,Ms] T[Zk,j,Bz]
6. T[U,cAf,Ms] T[amw,ZS,rK] T[flj,jxS,sAV] T[l,F,vU] T[Zk,j,Bz]
7. T[flj,jxS,sAV] T[Zk,j,Bz] T[l,F,vU] T[U,cAf,Ms] T[amw,ZS,rK]
8. T[flj,jxS,sAV] T[Zk,j,Bz] T[U,cAf,Ms] T[amw,ZS,rK] T[l,F,vU]
9. T[l,F,vU] T[U,cAf,Ms] T[flj,jxS,sAV] T[amw,ZS,rK] T[Zk,j,Bz]

Question 27
We consider a function f with 4 arguments, that are respectively in the sets D, A, B and D. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in D \times A \times B \times D$, in order to compute $f(x)$, we have to execute at most $O(|A| \cdot |B|^2 \cdot |D|)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|A|^3 \cdot |B|^2)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

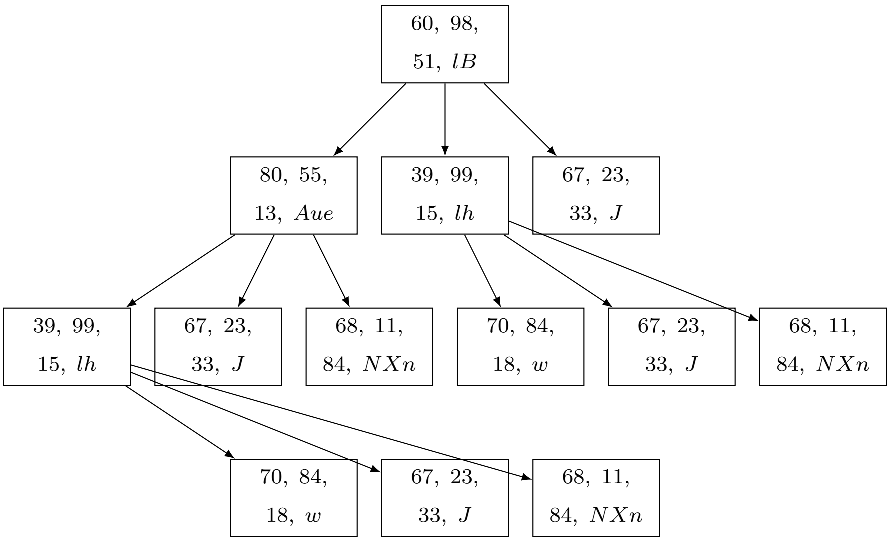
- 1 $O(|A|^4 \cdot |B|^4)$
- 2 $O(|A|^6 \cdot |B|^3 \cdot |D|^4)$
- 3 $O(|A|^3 \cdot |B|^6 \cdot |D|^4)$
- 4 $O(|A|^4 \cdot |B|^6 \cdot |D|)$
- 5 $O(|A|^4 \cdot |B|^3 \cdot |D|^3)$
- 6 $O(|A|^2 \cdot |B|^4 \cdot |D|^5)$
- 7 $O(|A|^3 \cdot |D|^5)$
- 8 $O(|A| \cdot |B|)$
- 9 $O(|A| \cdot |B|^5 \cdot |D|^5)$
- 10 $O(|A| \cdot |B| \cdot |D|^4)$

Question 28
Consider a function f with 3 arguments, 2 integers, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(12, 14, zSy)$. The recursive calls are done by following a depth-first search exploration of the tree going to the left before exploring the right part,
We assume that, when a state is final (when there is no recursive call), the computation is done in constant time. We also assume that, when the state is not tinal, the number of recursive call is not constant (it depends on the size of the input).
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. How many recursive calls of the function f_{MEMO} will be done in constant time?



[1] 2	[3] 4	[5] 8	[7] 5	[9] 14
[2] 13	[4] 7	[6] 6	[8] 12	[10] 3

Question 29
Consider a function f with 4 arguments, 3 integers, 1 string. This function is recursive. We draw hereinafter all the recursive calls that are done when we run $f(60, 98, 51, lB)$.
We would like to code this function using a dynamic programming algorithm, with the iterative version f_{ITER}. This function uses an array T to save the intermediate results. In which order the cells $T[39, 99, 15, lh], T[67, 23, 33, J], T[68, 11, 84, NXn], T[70, 84, 18, w], T[80, 55, 13, Aue]$ will be computed by f_{ITER}?



1. T[70,84,18,w] T[67,23,33,J] T[80,55,13,Aue] T[39,99,15,lh] T[68,11,84,NXn]
2. T[68,11,84,NXn] T[67,23,33,J] T[70,84,18,w] T[39,99,15,lh] T[80,55,13,Aue]
3. T[39,99,15,lh] T[70,84,18,w] T[68,11,84,NXn] T[80,55,13,Aue] T[67,23,33,J]
4. T[39,99,15,lh] T[68,11,84,NXn] T[70,84,18,w] T[80,55,13,Aue] T[67,23,33,J]
5. T[70,84,18,w] T[80,55,13,Aue] T[39,99,15,lh] T[67,23,33,J] T[68,11,84,NXn]
6. T[67,23,33,J] T[70,84,18,w] T[80,55,13,Aue] T[68,11,84,NXn] T[39,99,15,lh]
7. T[80,55,13,Aue] T[68,11,84,NXn] T[67,23,33,J] T[39,99,15,lh] T[70,84,18,w]
8. T[67,23,33,J] T[70,84,18,w] T[68,11,84,NXn] T[80,55,13,Aue] T[39,99,15,lh]
9. T[70,84,18,w] T[39,99,15,lh] T[68,11,84,NXn] T[80,55,13,Aue] T[67,23,33,J]

Question 30
We consider a function f with 3 arguments, that are respectively in the sets C, E and B. The size of those sets is finite. The function is recursive. We assume that, for any input $x \in C \times E \times B$, in order to compute $f(x)$, we have to execute at most $O(|B|^2)$ recursive calls ; and that, with the outputs of those calls, the function does a calculation in time $O(|C|^2 \cdot |E|^2)$.
Assuming we have coded this function with a fynamique programming algorithm, using the memoïzation version f_{MEMO}. What is the worst case complexity of f_{MEMO} ?

- [1] $O(|B|^5 \cdot |C| \cdot |E|^3)$
- [2] $O(|B|^6 \cdot |C|^6 \cdot |E|^5)$
- [3] $O(|B|^4 \cdot |C|^5 \cdot |E|^3)$
- [4] $O(|B|^3 \cdot |C|^3 \cdot |E|^3)$
- [5] $O(|B|^2 \cdot |C|^5 \cdot |E|^4)$
- [6] $O(|B|^6 \cdot |E|^5)$
- [7] $O(|B|^6 \cdot |C|^5 \cdot |E|^4)$
- [8] $O(|B|^6 \cdot |C|^3 \cdot |E|^6)$
- [9] $O(|B|^4 \cdot |C|^4 \cdot |E|^2)$
- [10] $O(|B|^2 \cdot |E|^5)$

Entraînement - Training

Noircissez complètement ci-dessous les 3 premières lettres de votre nom de famille et la première lettre de votre prénom. Par exemple, pour Jean Dupont, cochez J, D, U, P ; pour Henri Harley, cochez seulement H, A, R ; pour Bernard Ca, cochez seulement A, B, C.

Check entirely the 3 first letters of your lastname and the first letter of your firstname. For instance, for Jean Dupont, check J, D, U, P ; for Henri Harley, check only H, A, R ; for Bernard Ca, check only A, B, C.

A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

Then write your lastname and firstname below.

Nom et prénom :

....

Les réponses aux questions sont à donner exclusivement sur cette feuille. Les réponses données sur les feuilles précédentes ne seront pas prises en compte. Pour cocher une case, il faut la **noircir complètement**. Vous pouvez effacer votre réponse à la gomme ou avec du blanc, attention à ne pas effacer la case à cocher. Si vous êtes dans l'impossibilité de corriger une erreur, cette page est dupliquée au verso ; vous pouvez alors barrer cette feuille ci et répondre au verso.

Question 1 : 1 2 3 4 5 6 7 8 9 10

Question 2 : 1 2 3 4 5 6 7 8 9

Question 3 : 1 2 3 4 5 6 7 8 9 10

Question 4 : 1 2 3 4 5 6 7 8 9 10

Question 5 : 1 2 3 4 5 6 7 8 9

Question 6 : 1 2 3 4 5 6 7 8 9 10

Question 7 : 1 2 3 4 5 6 7 8 9 10

Question 8 : 1 2 3 4 5 6 7 8 9

Question 9 : 1 2 3 4 5 6 7 8 9 10

Question 10 : 1 2 3 4 5 6 7 8 9 10

Question 11 : 1 2 3 4 5 6 7 8 9

Question 12 : 1 2 3 4 5 6 7 8 9 10

Question 13 : 1 2 3 4 5 6 7 8 9 10

QUESTION 14 : 1 2 3 4 5 6 7 8 9

QUESTION 15 : 1 2 3 4 5 6 7 8 9 10

QUESTION 16 : 1 2 3 4 5 6 7 8 9 10

QUESTION 17 : 1 2 3 4 5 6 7 8 9

QUESTION 18 : 1 2 3 4 5 6 7 8 9 10

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QUESTION 22 : 1 2 3 4 5 6 7 8 9 10

QUESTION 23 : 1 2 3 4 5 6 7 8 9

QUESTION 24 : 1 2 3 4 5 6 7 8 9 10

QUESTION 25 : 1 2 3 4 5 6 7 8 9 10

QUESTION 26 : 1 2 3 4 5 6 7 8 9

QUESTION 27 : 1 2 3 4 5 6 7 8 9 10

QUESTION 28 : 1 2 3 4 5 6 7 8 9 10

QUESTION 29 : 1 2 3 4 5 6 7 8 9

QUESTION 30 : 1 2 3 4 5 6 7 8 9 10