



Entraînement - Training

INSTRUCTION : English version below

En haut de chaque page se trouvent 3 nombres, par exemple +1/3/58+. Vous **devez** vérifier que, sur chacune des pages de votre sujet, le **premier** de ces 3 nombres est le même (dans cet exemple, il s'agit donc du 1). Ce nombre identifie votre copie. Les deux autres nombres ne sont pas importants.

Détacher la dernière feuille et répondre dessus. Ne pas rendre les pages contenant les questions, vous ne devez rendre **que la dernière feuille**. Chaque question est sur 1 point, aucun point ne sera attribué aux questions contenant une mauvaise réponse.

Les questions faisant apparaître le symbole ♣ peuvent présenter une ou plusieurs bonnes réponses qui doivent toutes être cochées. Les autres ont une unique bonne réponse.

At the top of each page are written 3 numbers, +1/3/58+. You **must** check that, on each page you have, the **first** number is the same (in this case, it would be the number 1). This number is the id of your subject. The two other numbers are not important.

Answer only on the last page. Keep the other pages containing the questions, you just have to return **the last page**. Each right answer gives you 1 point. For any wrong answer, the mark of the question is 0.

If there is a question with a symbol ♣, there may be one or more right answer. All of them must be checked. Any other question has only one right answer.

Question 1 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0.53	0	0	0	0	0.47	0
S2	0.54	0.2	0	0	0	0	0	0.26
S3	0	0	0	0	0	0.87	0.13	0
S4	0	0	0	0.49	0	0	0.51	0
S5	0	0	0.79	0	0	0	0.21	0
S6	0	0	0.71	0	0.1	0.19	0	0
S7	0	0	0	0	0	1	0	0
S8	0	0	0	0	0	0	0	1

Check the transient states of the chain.

☐ S1

☐ S3

☐ S5

☐ S7

☐ S2

☐ S4

☐ S6

☐ S8



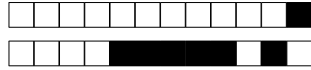
Question 2 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	1	0	0	0	0	0	0	0	0
S2	0	0.51	0.11	0.21	0	0.16	0	0.01	0	0
S3	0	0	0	0	0	1	0	0	0	0
S4	0.27	0	0	0	0	0.59	0	0	0	0.14
S5	0	0	0	0.94	0.05	0.01	0	0	0	0
S6	1	0	0	0	0	0	0	0	0	0
S7	0	0	0	0	1	0	0	0	0	0
S8	0	0	1	0	0	0	0	0	0	0
S9	0	0	0	0	0	0	0	1	0	0
S10	0	0	0.96	0	0	0.03	0	0	0.01	0

Check the recurrent states of the chain.

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------------------|
| ☐ 1 S1 | ☐ 3 S3 | ☐ 5 S5 | ☐ 7 S7 | ☐ 9 S9 |
| ☐ 2 S2 | ☐ 4 S4 | ☐ 6 S6 | ☐ 8 S8 | ☐ 10 S10 |

**Question 3 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.46	0.46	0	0	0	0	0	0	0.08	0
S2	0	0	0.6	0.4	0	0	0	0	0	0
S3	0	0	0	0	0	1	0	0	0	0
S4	0	0	1	0	0	0	0	0	0	0
S5	0	0	0	0	1	0	0	0	0	0
S6	0	0	0	0.95	0	0	0	0	0.05	0
S7	0	0	0	0	0.74	0	0.21	0	0.05	0
S8	0	0.69	0	0	0.31	0	0	0	0	0
S9	0	0	0	0	0	0	1	0	0	0
S10	0.38	0	0.38	0.23	0	0	0	0	0.01	0

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4



Question 4

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
S2	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
S3	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$
S4	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0
S5	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0
S6	0	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$
S7	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0
S8	0	1	0	0	0	0	0	0

What is the stationary distribution of the chain ?

- ☐ 1 (0.14 0.18 0.29 0.14 0.00 0.12 0.00 0.13)
- ☐ 2 (0.12 0.18 0.14 0.13 0.14 0.29 0.00 0.00)
- ☐ 3 (0.18 0.14 0.00 0.14 0.00 0.29 0.13 0.12)
- ☐ 4 (0.29 0.14 0.13 0.14 0.00 0.18 0.12 0.00)
- ☐ 5 (0.14 0.00 0.12 0.18 0.13 0.14 0.00 0.29)
- ☐ 6 (0.18 0.00 0.29 0.00 0.13 0.12 0.14 0.14)
- ☐ 7 (0.00 0.14 0.13 0.18 0.00 0.29 0.12 0.14)
- ☐ 8 (0.00 0.13 0.18 0.12 0.14 0.14 0.00 0.29)
- ☐ 9 Pas de distribution limite
- ☐ 10 (0.13 0.14 0.12 0.29 0.00 0.18 0.14 0.00)



Question 5 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0	0	0.77	0	0	0.23	0	0	0
S2	0	0.99	0	0	0	0	0	0	0.01
S3	0	0.94	0.05	0	0.01	0	0	0	0
S4	1	0	0	0	0	0	0	0	0
S5	0	0	0	0	0	0	0	1	0
S6	0	0	0.18	0.5	0.29	0	0	0	0.03
S7	0.48	0	0	0	0	0	0	0.25	0.27
S8	0	0	0	0	0	0	0	0	1
S9	0	0	0	0	1	0	0	0	0

Check the transient states of the chain.

☐ S1

☐ S3

☐ S5

☐ S7

☐ S9

☐ S2

☐ S4

☐ S6

☐ S8



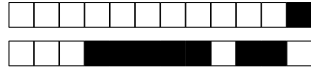
Question 6 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0	0	0	0.29	0.29	0	0	0.42	0
S2	0	0	0	0	0.96	0	0	0.02	0.02	0
S3	0	0	0	0	0	0.32	0	0.68	0	0
S4	0	0.16	0	0	0	0	0.32	0	0.28	0.24
S5	0	0	0	0.34	0.51	0	0.15	0	0	0
S6	0	0	0	0	0.86	0	0.14	0	0	0
S7	0	0	0	0	0	0	0.73	0.27	0	0
S8	0	0	0	0.9	0.04	0	0.06	0	0	0
S9	0	0	0	0	0	0	1	0	0	0
S10	0	0	0	0	0.9	0	0	0.02	0	0.08

Check the recurrent states of the chain.

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------------------|
| ☐ 1 S1 | ☐ 3 S3 | ☐ 5 S5 | ☐ 7 S7 | ☐ 9 S9 |
| ☐ 2 S2 | ☐ 4 S4 | ☐ 6 S6 | ☐ 8 S8 | ☐ 10 S10 |



Question 7 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0	0	0	0.46	0	0	0	0	0.54
S2	0	0	1	0	0	0	0	0	0	0
S3	0	1	0	0	0	0	0	0	0	0
S4	0	0.6	0	0	0	0.15	0.25	0	0	0
S5	0	0	0	0	0	1	0	0	0	0
S6	0	0	0.98	0.02	0	0	0	0	0	0
S7	0	0	0.1	0	0.13	0.77	0	0	0	0
S8	0	0.17	0.25	0.07	0.45	0.06	0	0	0	0
S9	0	0	0	0.19	0	0.33	0.26	0	0	0.22
S10	0	0.82	0	0	0	0.18	0	0	0	0

Check the period of each state.

- | | | | | |
|-----------------------------------|------------------------------------|------------------------------------|------------------------------------|-------------------------------------|
| ☐ 1 S1 : 1 | ☐ 9 S3 : 1 | ☐ 17 S5 : 1 | ☐ 25 S7 : 1 | ☐ 33 S9 : 1 |
| ☐ 2 S1 : 2 | ☐ 10 S3 : 2 | ☐ 18 S5 : 2 | ☐ 26 S7 : 2 | ☐ 34 S9 : 2 |
| ☐ 3 S1 : 3 | ☐ 11 S3 : 3 | ☐ 19 S5 : 3 | ☐ 27 S7 : 3 | ☐ 35 S9 : 3 |
| ☐ 4 S1 : 4 | ☐ 12 S3 : 4 | ☐ 20 S5 : 4 | ☐ 28 S7 : 4 | ☐ 36 S9 : 4 |
| ☐ 5 S2 : 1 | ☐ 13 S4 : 1 | ☐ 21 S6 : 1 | ☐ 29 S8 : 1 | ☐ 37 S10 : 1 |
| ☐ 6 S2 : 2 | ☐ 14 S4 : 2 | ☐ 22 S6 : 2 | ☐ 30 S8 : 2 | ☐ 38 S10 : 2 |
| ☐ 7 S2 : 3 | ☐ 15 S4 : 3 | ☐ 23 S6 : 3 | ☐ 31 S8 : 3 | ☐ 39 S10 : 3 |
| ☐ 8 S2 : 4 | ☐ 16 S4 : 4 | ☐ 24 S6 : 4 | ☐ 32 S8 : 4 | ☐ 40 S10 : 4 |

**Question 8**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	1	0	0	0	0	0
S2	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$
S3	0	0	0	0	0	1	0	0
S4	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0	0
S5	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
S6	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0
S7	0	0	0	1	0	0	0	0
S8	$\frac{1}{4}$	0	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0

What is the stationary distribution of the chain ?

- ☐ 1 (0.00 0.09 0.09 0.26 0.17 0.00 0.35 0.04)
- ☐ 2 (0.17 0.00 0.00 0.04 0.35 0.09 0.26 0.09)
- ☐ 3 (0.26 0.00 0.04 0.35 0.17 0.09 0.00 0.09)
- ☐ 4 Pas de distribution limite
- ☐ 5 (0.04 0.26 0.17 0.09 0.35 0.00 0.00 0.09)
- ☐ 6 (0.09 0.00 0.04 0.35 0.17 0.00 0.09 0.26)
- ☐ 7 (0.00 0.26 0.09 0.35 0.09 0.17 0.04 0.00)
- ☐ 8 (0.09 0.09 0.17 0.00 0.26 0.00 0.04 0.35)
- ☐ 9 (0.04 0.00 0.26 0.09 0.17 0.35 0.09 0.00)
- ☐ 10 (0.17 0.35 0.09 0.00 0.26 0.09 0.04 0.00)



Question 9 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0	0	0	0.81	0.19	0	0	0	0
S2	0	0.98	0	0	0.02	0	0	0	0
S3	0	0.75	0	0.13	0	0.12	0	0	0
S4	0	0	0	0	0.78	0.09	0.13	0	0
S5	0	0	0	0.9	0.02	0	0.03	0.05	0
S6	0	0.64	0	0	0.01	0.18	0	0.07	0.1
S7	0	0	0	0	0	0	0	1	0
S8	0	0	0	0	0	1	0	0	0
S9	0	0	0	0	0.97	0.03	0	0	0

Check the transient states of the chain.

☐ 1 S1

☐ 3 S3

☐ 5 S5

☐ 7 S7

☐ 9 S9

☐ 2 S2

☐ 4 S4

☐ 6 S6

☐ 8 S8



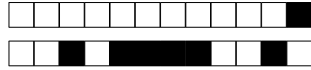
Question 10 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.49	0	0	0	0.51	0	0	0	0	0
S2	0	0	0	0	0	0.26	0.65	0	0	0.09
S3	0	0.81	0	0	0.03	0.09	0	0.07	0	0
S4	0	0	0	0	0	0	0	0	1	0
S5	0	0	0	0.49	0.4	0	0	0	0.11	0
S6	0	0.27	0	0	0	0	0	0.7	0.03	0
S7	0	0	0.7	0	0	0	0	0	0.14	0.16
S8	0.49	0.51	0	0	0	0	0	0	0	0
S9	0	0	0	1	0	0	0	0	0	0
S10	0	0	0	0.13	0	0.5	0.06	0.31	0	0

Check the recurrent states of the chain.

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------------------|
| ☐ 1 S1 | ☐ 3 S3 | ☐ 5 S5 | ☐ 7 S7 | ☐ 9 S9 |
| ☐ 2 S2 | ☐ 4 S4 | ☐ 6 S6 | ☐ 8 S8 | ☐ 10 S10 |

**Question 11 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0	0	0	1	0	0	0	0	0
S2	0.43	0	0	0.15	0	0.26	0.11	0	0	0.05
S3	0	0	0	0.56	0	0	0	0.31	0.13	0
S4	1	0	0	0	0	0	0	0	0	0
S5	0	0	1	0	0	0	0	0	0	0
S6	0	0	0.62	0.16	0	0.21	0	0	0	0.01
S7	0	0.25	0	0	0.33	0	0	0.31	0	0.11
S8	0	0	0	0	0	1	0	0	0	0
S9	0	0	0	0.53	0.35	0.12	0	0	0	0
S10	0.7	0	0	0.27	0	0	0	0	0	0.03

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4

**Question 12**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
S2	1	0	0	0	0	0	0	0
S3	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	0
S4	0	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
S5	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$	0
S6	0	1	0	0	0	0	0	0
S7	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
S8	0	1	0	0	0	0	0	0

What is the stationary distribution of the chain ?

- ☐ 1 (0.27 0.13 0.00 0.07 0.13 0.27 0.00 0.13)
- ☐ 2 (0.07 0.13 0.27 0.27 0.00 0.13 0.00 0.13)
- ☐ 3 Pas de distribution limite
- ☐ 4 (0.13 0.00 0.13 0.27 0.07 0.13 0.00 0.27)
- ☐ 5 (0.27 0.27 0.13 0.00 0.00 0.13 0.13 0.07)
- ☐ 6 (0.13 0.07 0.13 0.27 0.00 0.27 0.13 0.00)
- ☐ 7 (0.13 0.07 0.13 0.27 0.00 0.27 0.13 0.00)
- ☐ 8 (0.27 0.07 0.13 0.27 0.13 0.00 0.13 0.00)
- ☐ 9 (0.13 0.27 0.13 0.07 0.13 0.27 0.00 0.00)
- ☐ 10 (0.13 0.00 0.13 0.13 0.27 0.00 0.27 0.07)



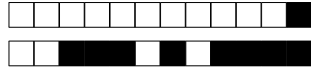
Question 13 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0	0	0	0.07	0	0.49	0	0.44	0
S2	0	0.14	0.77	0.06	0	0	0.03	0	0	0
S3	0	0	0	0.72	0.06	0	0.05	0	0.17	0
S4	0	0.17	0.31	0.09	0	0.4	0.03	0	0	0
S5	0.22	0	0	0	0.78	0	0	0	0	0
S6	0.08	0	0	0	0.2	0	0.69	0	0.03	0
S7	0.64	0	0	0.13	0	0.23	0	0	0	0
S8	0	0	0	0	0	0	0.75	0.25	0	0
S9	0.43	0.51	0	0.04	0.02	0	0	0	0	0
S10	0	0	0	0	0	0	0	1	0	0

Check the transient states of the chain.

- | | | | | | | | | | |
|----------------------------|----|----------------------------|----|----------------------------|----|----------------------------|----|-----------------------------|-----|
| ☐ 1 | S1 | ☐ 3 | S3 | ☐ 5 | S5 | ☐ 7 | S7 | ☐ 9 | S9 |
| ☐ 2 | S2 | ☐ 4 | S4 | ☐ 6 | S6 | ☐ 8 | S8 | ☐ 10 | S10 |



Question 14 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.66	0	0	0	0	0	0.34	0	0	0
S2	0.16	0.8	0	0	0	0.01	0.03	0	0	0
S3	0	0.96	0	0	0	0	0	0.04	0	0
S4	1	0	0	0	0	0	0	0	0	0
S5	0.07	0.12	0.09	0	0.39	0.32	0	0.01	0	0
S6	0	0	0	0	0	0	1	0	0	0
S7	0	0	0	0	0	0	0	0.96	0.04	0
S8	0.72	0	0	0	0	0.18	0.08	0	0.02	0
S9	0.16	0.84	0	0	0	0	0	0	0	0
S10	0.86	0	0	0.14	0	0	0	0	0	0

Check the recurrent states of the chain.

- | | | | | | | | | | |
|----------------------------|----|----------------------------|----|----------------------------|----|----------------------------|----|-----------------------------|-----|
| ☐ 1 | S1 | ☐ 3 | S3 | ☐ 5 | S5 | ☐ 7 | S7 | ☐ 9 | S9 |
| ☐ 2 | S2 | ☐ 4 | S4 | ☐ 6 | S6 | ☐ 8 | S8 | ☐ 10 | S10 |

**Question 15 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.65	0	0	0.18	0.17	0	0	0	0	0
S2	0	0	0	0	0.4	0.57	0	0	0.03	0
S3	0	0	0	0	0	0	0	0	0	1
S4	0	0	0	0.32	0	0.65	0	0.03	0	0
S5	0	0.6	0	0	0	0.37	0	0	0.03	0
S6	0	0	0	0	0	1	0	0	0	0
S7	0	0	0.34	0	0	0.07	0	0.59	0	0
S8	0	0	0	0.45	0	0.55	0	0	0	0
S9	0	0	0.97	0.03	0	0	0	0	0	0
S10	0	0	0	1	0	0	0	0	0	0

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4



Question 16

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	0	$\frac{1}{3}$
S2	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	0
S3	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0
S4	0	0	0	0	1	0	0	0
S5	0	0	0	0	$\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{1}{4}$
S6	0	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0
S7	1	0	0	0	0	0	0	0
S8	0	$\frac{1}{4}$	0	$\frac{1}{2}$	0	0	$\frac{1}{4}$	0

What is the stationary distribution of the chain ?

- ☐ 1 (0.21 0.22 0.22 0.00 0.12 0.06 0.00 0.18)
- ☐ 2 (0.21 0.00 0.00 0.22 0.06 0.22 0.18 0.12)
- ☐ 3 (0.12 0.18 0.22 0.00 0.06 0.00 0.21 0.22)
- ☐ 4 (0.22 0.00 0.00 0.06 0.22 0.12 0.21 0.18)
- ☐ 5 (0.00 0.12 0.18 0.06 0.00 0.21 0.22 0.22)
- ☐ 6 (0.22 0.06 0.00 0.22 0.21 0.18 0.12 0.00)
- ☐ 7 (0.22 0.00 0.12 0.06 0.18 0.21 0.22 0.00)
- ☐ 8 (0.22 0.21 0.00 0.06 0.18 0.00 0.22 0.12)
- ☐ 9 Pas de distribution limite
- ☐ 10 (0.00 0.06 0.21 0.00 0.18 0.12 0.22 0.22)



Question 17 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0	0.65	0	0	0	0.35	0	0	0
S2	0	0	0	0	0	1	0	0	0
S3	0	0	0	0	0	0.55	0	0.45	0
S4	0	0.48	0	0.34	0	0.18	0	0	0
S5	0	0	1	0	0	0	0	0	0
S6	0	0.94	0	0	0	0.06	0	0	0
S7	0	0	0.8	0.11	0	0	0	0	0.09
S8	1	0	0	0	0	0	0	0	0
S9	0	0	0	0	0	0	1	0	0

Check the transient states of the chain.

☐ S1

☐ S3

☐ S5

☐ S7

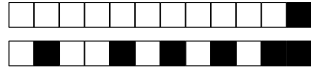
☐ S9

☐ S2

☐ S4

☐ S6

☐ S8



Question 18 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	1	0	0	0	0	0	0	0	0
S2	0	1	0	0	0	0	0	0	0
S3	0	0	0	0	0	0	0	0	1
S4	0	0.42	0	0	0	0	0.28	0.12	0.18
S5	0	0	0.89	0.07	0	0	0	0.04	0
S6	0	0.73	0.23	0	0	0.04	0	0	0
S7	0	0	0	0	0	0	0	1	0
S8	0.36	0	0	0	0	0	0.46	0	0.18
S9	0.65	0.12	0	0	0.23	0	0	0	0

Check the recurrent states of the chain.

☐ S1

☐ S3

☐ S5

☐ S7

☐ S9

☐ S2

☐ S4

☐ S6

☐ S8



Question 19 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.25	0	0	0.75	0	0	0	0	0	0
S2	0	0	0	0	0	0	1	0	0	0
S3	0.72	0	0	0	0	0	0	0.28	0	0
S4	0	0	0.13	0.67	0	0	0	0.2	0	0
S5	0	0	0	0	1	0	0	0	0	0
S6	0	0	0	0	0	0.45	0	0	0	0.55
S7	0	0	0	0	0	1	0	0	0	0
S8	0	0	1	0	0	0	0	0	0	0
S9	0	0	0	0	0.47	0	0	0.31	0	0.22
S10	0.57	0	0	0	0	0	0	0.23	0.2	0

Check the period of each state.

- | | | | | |
|-------------------------------|--------------------------------|--------------------------------|--------------------------------|---------------------------------|
| <input type="text"/> 1 S1 : 1 | <input type="text"/> 9 S3 : 1 | <input type="text"/> 17 S5 : 1 | <input type="text"/> 25 S7 : 1 | <input type="text"/> 33 S9 : 1 |
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| <input type="text"/> 3 S1 : 3 | <input type="text"/> 11 S3 : 3 | <input type="text"/> 19 S5 : 3 | <input type="text"/> 27 S7 : 3 | <input type="text"/> 35 S9 : 3 |
| <input type="text"/> 4 S1 : 4 | <input type="text"/> 12 S3 : 4 | <input type="text"/> 20 S5 : 4 | <input type="text"/> 28 S7 : 4 | <input type="text"/> 36 S9 : 4 |
| <input type="text"/> 5 S2 : 1 | <input type="text"/> 13 S4 : 1 | <input type="text"/> 21 S6 : 1 | <input type="text"/> 29 S8 : 1 | <input type="text"/> 37 S10 : 1 |
| <input type="text"/> 6 S2 : 2 | <input type="text"/> 14 S4 : 2 | <input type="text"/> 22 S6 : 2 | <input type="text"/> 30 S8 : 2 | <input type="text"/> 38 S10 : 2 |
| <input type="text"/> 7 S2 : 3 | <input type="text"/> 15 S4 : 3 | <input type="text"/> 23 S6 : 3 | <input type="text"/> 31 S8 : 3 | <input type="text"/> 39 S10 : 3 |
| <input type="text"/> 8 S2 : 4 | <input type="text"/> 16 S4 : 4 | <input type="text"/> 24 S6 : 4 | <input type="text"/> 32 S8 : 4 | <input type="text"/> 40 S10 : 4 |



Question 20

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	0	0	0	0	0	1
S2	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	0
S3	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	0
S4	0	0	0	0	0	1	0	0
S5	0	0	0	0	1	0	0	0
S6	$\frac{1}{3}$	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$
S7	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	0
S8	0	0	0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$

What is the stationary distribution of the chain ?

- ☐ 1 (0.09 0.31 0.09 0.10 0.15 0.00 0.00 0.26)
- ☐ 2 (0.09 0.31 0.15 0.26 0.10 0.09 0.00 0.00)
- ☐ 3 (0.09 0.09 0.00 0.26 0.00 0.31 0.10 0.15)
- ☐ 4 (0.26 0.15 0.10 0.09 0.09 0.00 0.31 0.00)
- ☐ 5 (0.09 0.09 0.10 0.00 0.15 0.31 0.00 0.26)
- ☐ 6 (0.26 0.00 0.15 0.09 0.10 0.09 0.00 0.31)
- ☐ 7 (0.10 0.00 0.09 0.15 0.26 0.00 0.09 0.31)
- ☐ 8 (0.00 0.00 0.26 0.09 0.15 0.31 0.09 0.10)
- ☐ 9 (0.10 0.31 0.00 0.15 0.26 0.00 0.09 0.09)
- ☐ 10 Pas de distribution limite



Question 21 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0.51	0	0	0.41	0	0.05	0.03	0	0
S2	0	0	0	0	1	0	0	0	0
S3	0.91	0	0	0.09	0	0	0	0	0
S4	1	0	0	0	0	0	0	0	0
S5	0.04	0	0.5	0	0.46	0	0	0	0
S6	0	0	0	0	0	0	1	0	0
S7	0.03	0.94	0	0	0	0	0.03	0	0
S8	0	0	1	0	0	0	0	0	0
S9	0.23	0	0	0	0	0	0	0.77	0

Check the transient states of the chain.

☐ 1 S1

☐ 3 S3

☐ 5 S5

☐ 7 S7

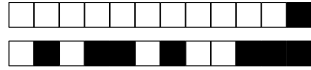
☐ 9 S9

☐ 2 S2

☐ 4 S4

☐ 6 S6

☐ 8 S8



Question 22 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0.89	0	0	0.11	0	0	0
S2	0	0.46	0	0	0.13	0	0	0.41
S3	0.57	0.37	0	0	0	0	0.06	0
S4	0	0	0	0	1	0	0	0
S5	0	0	0	1	0	0	0	0
S6	0	1	0	0	0	0	0	0
S7	0.18	0.82	0	0	0	0	0	0
S8	0.31	0	0	0	0	0.69	0	0

Check the recurrent states of the chain.

- | | | | | | | | |
|----------------------------|----|----------------------------|----|----------------------------|----|----------------------------|----|
| ☐ 1 | S1 | ☐ 3 | S3 | ☐ 5 | S5 | ☐ 7 | S7 |
| ☐ 2 | S2 | ☐ 4 | S4 | ☐ 6 | S6 | ☐ 8 | S8 |

**Question 23 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0.2	0	0	0.8	0	0	0	0	0
S2	0	0	0	0.98	0	0.02	0	0	0	0
S3	0	0	0	0.76	0	0	0	0	0.16	0.08
S4	0.05	0	0	0.48	0.18	0	0	0	0	0.29
S5	0	0.36	0	0.64	0	0	0	0	0	0
S6	0	0	0	1	0	0	0	0	0	0
S7	0	0.24	0.54	0	0	0.22	0	0	0	0
S8	0	0	0	0	0.64	0	0.36	0	0	0
S9	0	0	0	0	0.5	0	0	0.5	0	0
S10	0	0.86	0	0	0	0.14	0	0	0	0

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4

**Question 24**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
S2	0	0	0	0	0	0	1	0
S3	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	0
S4	0	$\frac{1}{4}$	0	$\frac{1}{2}$	0	0	0	$\frac{1}{4}$
S5	0	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	0
S6	$\frac{1}{4}$	0	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	0	0
S7	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	0
S8	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0	0	0	$\frac{1}{2}$

What is the stationary distribution of the chain ?

- ☐ 1 (0.06 0.30 0.16 0.24 0.00 0.14 0.00 0.10)
- ☐ 2 (0.10 0.14 0.00 0.30 0.00 0.06 0.16 0.24)
- ☐ 3 (0.16 0.10 0.00 0.06 0.24 0.00 0.30 0.14)
- ☐ 4 (0.16 0.10 0.06 0.00 0.14 0.24 0.30 0.00)
- ☐ 5 (0.14 0.24 0.16 0.10 0.06 0.30 0.00 0.00)
- ☐ 6 (0.30 0.14 0.00 0.10 0.00 0.06 0.24 0.16)
- ☐ 7 (0.06 0.10 0.24 0.16 0.30 0.00 0.14 0.00)
- ☐ 8 (0.30 0.24 0.00 0.14 0.00 0.16 0.10 0.06)
- ☐ 9 (0.00 0.14 0.00 0.10 0.06 0.30 0.24 0.16)
- ☐ 10 Pas de distribution limite



Question 25 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0	0	0.13	0.87	0	0	0	0	0
S2	0	0	0.58	0.39	0	0	0.03	0	0
S3	1	0	0	0	0	0	0	0	0
S4	0	0	0.96	0	0	0	0.04	0	0
S5	0.75	0	0.25	0	0	0	0	0	0
S6	0	0.54	0	0	0	0	0	0.37	0.09
S7	0	1	0	0	0	0	0	0	0
S8	0	0	0	0	1	0	0	0	0
S9	0.48	0.17	0	0	0.26	0.05	0	0	0.04

Check the transient states of the chain.

☐ 1 S1

☐ 3 S3

☐ 5 S5

☐ 7 S7

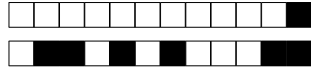
☐ 9 S9

☐ 2 S2

☐ 4 S4

☐ 6 S6

☐ 8 S8



Question 26 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0	0.93	0	0	0	0.02	0	0.05	0
S2	0.38	0.29	0	0	0.33	0	0	0	0
S3	1	0	0	0	0	0	0	0	0
S4	0	0	1	0	0	0	0	0	0
S5	0	0.73	0	0	0	0.27	0	0	0
S6	0	0.12	0.46	0.15	0	0.27	0	0	0
S7	0	0	0	0	0	0	0.81	0.19	0
S8	0.62	0	0	0	0.38	0	0	0	0
S9	0	0	0.53	0	0.34	0	0	0	0.13

Check the recurrent states of the chain.

☐ S1

☐ S3

☐ S5

☐ S7

☐ S9

☐ S2

☐ S4

☐ S6

☐ S8

**Question 27 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.34	0	0	0	0	0	0	0.66	0	0
S2	0	0	0.57	0	0	0	0	0	0	0.43
S3	0	0	0.44	0	0	0.47	0	0.09	0	0
S4	0	0	0.86	0.09	0	0.02	0.03	0	0	0
S5	1	0	0	0	0	0	0	0	0	0
S6	0	0	0	0	0	0	1	0	0	0
S7	0	0	0	1	0	0	0	0	0	0
S8	0	0	0	0	0	0	0	1	0	0
S9	0	0	0.58	0	0	0	0.42	0	0	0
S10	0.99	0.01	0	0	0	0	0	0	0	0

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4



Question 28

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	0	0	0	0	0	1
S2	0	1	0	0	0	0	0	0
S3	0	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0
S4	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	0	0	$\frac{1}{2}$
S5	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$
S6	0	0	0	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$
S7	0	0	0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$
S8	0	0	0	0	0	1	0	0

What is the stationary distribution of the chain ?

- ☐ 1 (0.00 0.21 0.12 0.08 0.21 0.16 0.22 0.00)
- ☐ 2 (0.00 0.22 0.21 0.12 0.08 0.16 0.00 0.21)
- ☐ 3 (0.16 0.21 0.00 0.21 0.12 0.08 0.22 0.00)
- ☐ 4 (0.21 0.22 0.21 0.12 0.16 0.00 0.00 0.08)
- ☐ 5 (0.12 0.22 0.21 0.16 0.21 0.00 0.00 0.08)
- ☐ 6 (0.12 0.21 0.22 0.16 0.21 0.00 0.00 0.08)
- ☐ 7 Pas de distribution limite
- ☐ 8 (0.22 0.21 0.21 0.00 0.16 0.00 0.12 0.08)
- ☐ 9 (0.22 0.12 0.00 0.00 0.08 0.21 0.21 0.16)
- ☐ 10 (0.00 0.21 0.16 0.21 0.12 0.00 0.08 0.22)



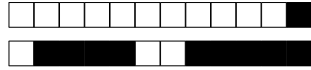
Question 29 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0.62	0.05	0	0	0.25	0	0	0	0.08
S2	0	0	0.18	0	0	0	0.17	0.31	0	0.34
S3	0.32	0	0	0	0	0	0	0.68	0	0
S4	0.67	0	0	0	0.32	0.01	0	0	0	0
S5	0	0	0.25	0.75	0	0	0	0	0	0
S6	0.76	0	0	0	0	0.06	0.13	0.04	0.01	0
S7	0	0	1	0	0	0	0	0	0	0
S8	0	0.92	0.06	0	0	0	0	0	0.02	0
S9	0	0	0.84	0	0	0.12	0	0.03	0	0.01
S10	0	0	1	0	0	0	0	0	0	0

Check the transient states of the chain.

- | | | | | | | | | | |
|----------------------------|----|----------------------------|----|----------------------------|----|----------------------------|----|-----------------------------|-----|
| ☐ 1 | S1 | ☐ 3 | S3 | ☐ 5 | S5 | ☐ 7 | S7 | ☐ 9 | S9 |
| ☐ 2 | S2 | ☐ 4 | S4 | ☐ 6 | S6 | ☐ 8 | S8 | ☐ 10 | S10 |



Question 30 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0.38	0	0.33	0	0.24	0	0.05	0	0
S2	0	0.38	0	0	0	0	0.62	0	0
S3	0.37	0	0.31	0	0	0	0.32	0	0
S4	0	0	0	0.32	0.53	0	0	0.15	0
S5	0	0	0.3	0	0	0	0.42	0.28	0
S6	0	0.15	0.76	0	0	0	0	0.02	0.07
S7	0	0	0	0	1	0	0	0	0
S8	0.95	0	0	0.05	0	0	0	0	0
S9	0	0	0	0	0	0.44	0	0	0.56

Check the recurrent states of the chain.

☐ S1

☐ S3

☐ S5

☐ S7

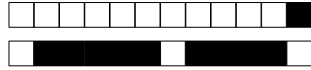
☐ S9

☐ S2

☐ S4

☐ S6

☐ S8

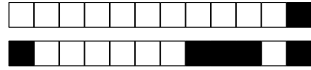
**Question 31 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	1	0	0	0	0	0	0	0	0	0
S2	0	0	0	0	0	0	0	0	1	0
S3	0	0.34	0	0.29	0	0	0	0.37	0	0
S4	0	0	0	0	0.18	0.82	0	0	0	0
S5	0	0.13	0	0	0	0	0.8	0.04	0	0.03
S6	0	0	0	0	0.65	0	0	0	0.35	0
S7	0	0	1	0	0	0	0	0	0	0
S8	0.4	0	0	0	0	0.13	0	0.47	0	0
S9	0	1	0	0	0	0	0	0	0	0
S10	0	0.28	0	0	0	0	0	0.41	0.31	0

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4



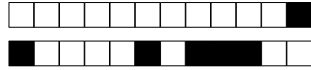
Question 32

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	1	0	0	0	0	0
S2	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0
S3	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0	0	$\frac{1}{2}$	0
S4	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0
S5	0	0	0	1	0	0	0	0
S6	0	0	1	0	0	0	0	0
S7	$\frac{1}{4}$	0	0	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0
S8	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	0

What is the stationary distribution of the chain ?

- ☐ 1 (0.00 0.13 0.10 0.21 0.31 0.00 0.21 0.05)
- ☐ 2 (0.00 0.10 0.31 0.21 0.21 0.05 0.00 0.13)
- ☐ 3 (0.13 0.00 0.31 0.21 0.05 0.10 0.21 0.00)
- ☐ 4 (0.21 0.00 0.00 0.13 0.10 0.21 0.31 0.05)
- ☐ 5 (0.00 0.00 0.13 0.05 0.21 0.21 0.31 0.10)
- ☐ 6 (0.13 0.10 0.21 0.31 0.21 0.00 0.05 0.00)
- ☐ 7 (0.10 0.00 0.21 0.00 0.05 0.21 0.13 0.31)
- ☐ 8 Pas de distribution limite
- ☐ 9 (0.05 0.31 0.10 0.00 0.21 0.00 0.13 0.21)
- ☐ 10 (0.13 0.10 0.00 0.21 0.21 0.00 0.31 0.05)



Question 33 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9
S1	0	0.93	0	0	0	0.02	0	0	0.05
S2	0	0	0	0	0	1	0	0	0
S3	1	0	0	0	0	0	0	0	0
S4	0	0	0	0	1	0	0	0	0
S5	1	0	0	0	0	0	0	0	0
S6	0	0.33	0.27	0.1	0	0.25	0	0	0.05
S7	0.2	0	0	0.78	0	0.01	0.01	0	0
S8	0	0	0	0	0	0	0	0	1
S9	0	0	0	0	1	0	0	0	0

Check the transient states of the chain.

☐ S1

☐ S3

☐ S5

☐ S7

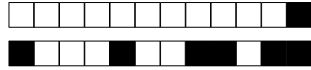
☐ S9

☐ S2

☐ S4

☐ S6

☐ S8



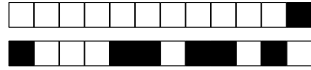
Question 34 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.26	0.13	0.15	0	0	0.18	0	0.17	0	0.11
S2	0	0	0	0	0	0	0	0.66	0.34	0
S3	0	0	0	0	0	0	0	1	0	0
S4	0	0	0.83	0.17	0	0	0	0	0	0
S5	0	0	0	0.62	0	0.36	0.02	0	0	0
S6	0	0.13	0.41	0	0	0	0.43	0.03	0	0
S7	0	0	0	0	0	0	0	0	1	0
S8	0	0	0.91	0	0	0.09	0	0	0	0
S9	0	0	0	0	0.36	0	0	0.64	0	0
S10	0	0.29	0	0	0	0.54	0	0.17	0	0

Check the recurrent states of the chain.

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------------------|
| ☐ 1 S1 | ☐ 3 S3 | ☐ 5 S5 | ☐ 7 S7 | ☐ 9 S9 |
| ☐ 2 S2 | ☐ 4 S4 | ☐ 6 S6 | ☐ 8 S8 | ☐ 10 S10 |

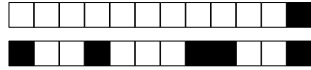
**Question 35 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0	0	0	0	1	0	0	0	0
S2	1	0	0	0	0	0	0	0	0	0
S3	0	0	0	0	0	0	0	0	1	0
S4	0.31	0	0	0	0	0	0	0	0	0.69
S5	0.75	0.12	0	0	0	0	0	0	0.13	0
S6	0	0	0	0.36	0	0	0.64	0	0	0
S7	0.87	0.01	0.12	0	0	0	0	0	0	0
S8	0	0	0	0	0.41	0	0.49	0	0.1	0
S9	0	0	1	0	0	0	0	0	0	0
S10	0	0	0	0	0	1	0	0	0	0

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4



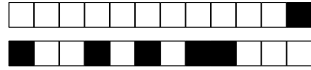
Question 36

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
S2	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
S3	0	0	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	$\frac{1}{3}$
S4	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	$\frac{1}{2}$	0	0
S5	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
S6	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$
S7	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	0	0
S8	0	1	0	0	0	0	0	0

What is the stationary distribution of the chain ?

- ☐ 1 (0.00 0.07 0.14 0.13 0.17 0.00 0.23 0.27)
- ☐ 2 (0.23 0.27 0.17 0.13 0.00 0.07 0.14 0.00)
- ☐ 3 (0.23 0.17 0.07 0.27 0.00 0.13 0.14 0.00)
- ☐ 4 (0.13 0.27 0.17 0.14 0.00 0.07 0.00 0.23)
- ☐ 5 (0.00 0.14 0.23 0.13 0.00 0.17 0.27 0.07)
- ☐ 6 Pas de distribution limite
- ☐ 7 (0.13 0.27 0.07 0.00 0.00 0.14 0.17 0.23)
- ☐ 8 (0.27 0.00 0.07 0.23 0.14 0.17 0.00 0.13)
- ☐ 9 (0.14 0.00 0.17 0.27 0.23 0.00 0.07 0.13)
- ☐ 10 (0.14 0.00 0.27 0.23 0.17 0.13 0.00 0.07)



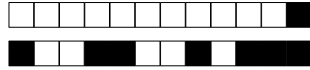
Question 37 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.81	0	0	0	0.19	0	0	0	0	0
S2	0.68	0.08	0	0.21	0	0	0	0	0	0.03
S3	0	0	0.9	0	0	0	0	0.1	0	0
S4	0.47	0	0	0	0	0	0.19	0.34	0	0
S5	1	0	0	0	0	0	0	0	0	0
S6	0.49	0.24	0.11	0	0.08	0	0	0	0	0.08
S7	0	0	0	0.66	0.02	0	0	0.15	0.04	0.13
S8	0	0	1	0	0	0	0	0	0	0
S9	0	0.92	0	0	0	0	0.03	0	0	0.05
S10	0.45	0	0	0	0	0	0.39	0	0.16	0

Check the transient states of the chain.

- | | | | | | | | | | |
|----------------------------|----|----------------------------|----|----------------------------|----|----------------------------|----|-----------------------------|-----|
| ☐ 1 | S1 | ☐ 3 | S3 | ☐ 5 | S5 | ☐ 7 | S7 | ☐ 9 | S9 |
| ☐ 2 | S2 | ☐ 4 | S4 | ☐ 6 | S6 | ☐ 8 | S8 | ☐ 10 | S10 |



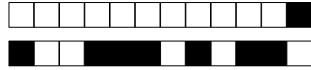
Question 38 ♣

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0	0	0	0.66	0	0	0	0.34	0	0
S2	0	0.74	0.16	0	0.1	0	0	0	0	0
S3	0	0	0.08	0	0.75	0	0.17	0	0	0
S4	0.79	0	0	0	0	0	0.21	0	0	0
S5	0	0	0	0	0	0.53	0	0	0.47	0
S6	0.45	0	0	0.3	0	0.09	0.1	0	0.06	0
S7	0	0	0	0	0	0.55	0.45	0	0	0
S8	0	0	0.32	0	0	0.68	0	0	0	0
S9	0	0	0.11	0	0.85	0	0	0.04	0	0
S10	0	0	0	0	0.61	0.36	0	0	0.03	0

Check the recurrent states of the chain.

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------------------|
| ☐ 1 S1 | ☐ 3 S3 | ☐ 5 S5 | ☐ 7 S7 | ☐ 9 S9 |
| ☐ 2 S2 | ☐ 4 S4 | ☐ 6 S6 | ☐ 8 S8 | ☐ 10 S10 |

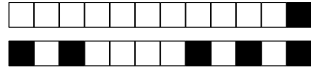
**Question 39 ♣**

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.23	0.46	0.13	0	0	0.09	0.06	0	0	0.03
S2	0.94	0	0.03	0	0	0	0.03	0	0	0
S3	0	0	0	0	0	0.7	0.14	0	0.16	0
S4	0.97	0	0	0.01	0	0	0.02	0	0	0
S5	0.03	0.75	0	0	0.22	0	0	0	0	0
S6	0	0	0	0	0	0	0	0	1	0
S7	0	0.33	0	0.59	0	0	0	0	0.06	0.02
S8	0	0	0	0	1	0	0	0	0	0
S9	0	0	0	0	0	1	0	0	0	0
S10	0	0	0	0.98	0.02	0	0	0	0	0

Check the period of each state.

<u>1</u> S1 : 1	<u>9</u> S3 : 1	<u>17</u> S5 : 1	<u>25</u> S7 : 1	<u>33</u> S9 : 1
<u>2</u> S1 : 2	<u>10</u> S3 : 2	<u>18</u> S5 : 2	<u>26</u> S7 : 2	<u>34</u> S9 : 2
<u>3</u> S1 : 3	<u>11</u> S3 : 3	<u>19</u> S5 : 3	<u>27</u> S7 : 3	<u>35</u> S9 : 3
<u>4</u> S1 : 4	<u>12</u> S3 : 4	<u>20</u> S5 : 4	<u>28</u> S7 : 4	<u>36</u> S9 : 4
<u>5</u> S2 : 1	<u>13</u> S4 : 1	<u>21</u> S6 : 1	<u>29</u> S8 : 1	<u>37</u> S10 : 1
<u>6</u> S2 : 2	<u>14</u> S4 : 2	<u>22</u> S6 : 2	<u>30</u> S8 : 2	<u>38</u> S10 : 2
<u>7</u> S2 : 3	<u>15</u> S4 : 3	<u>23</u> S6 : 3	<u>31</u> S8 : 3	<u>39</u> S10 : 3
<u>8</u> S2 : 4	<u>16</u> S4 : 4	<u>24</u> S6 : 4	<u>32</u> S8 : 4	<u>40</u> S10 : 4



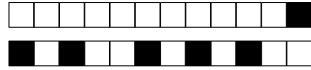
Question 40

We consider a Markov chain with the following transition matrix P :

P_{ij}	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	$\frac{1}{4}$	0	0	$\frac{1}{2}$	0	0	$\frac{1}{4}$
S2	0	0	0	0	0	1	0	0
S3	0	0	1	0	0	0	0	0
S4	0	0	0	0	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$
S5	0	0	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$
S6	0	0	0	0	0	0	0	1
S7	$\frac{1}{4}$	0	0	0	$\frac{1}{2}$	$\frac{1}{4}$	0	0
S8	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	0	$\frac{1}{3}$

What is the stationary distribution of the chain ?

- ☐ 1 (0.25 0.09 0.26 0.11 0.00 0.03 0.00 0.26)
- ☐ 2 (0.26 0.11 0.26 0.00 0.03 0.09 0.25 0.00)
- ☐ 3 (0.26 0.09 0.00 0.25 0.03 0.11 0.26 0.00)
- ☐ 4 (0.09 0.00 0.26 0.25 0.03 0.11 0.00 0.26)
- ☐ 5 (0.26 0.11 0.25 0.00 0.09 0.03 0.00 0.26)
- ☐ 6 (0.00 0.03 0.09 0.25 0.26 0.11 0.26 0.00)
- ☐ 7 (0.03 0.11 0.26 0.09 0.00 0.00 0.26 0.25)
- ☐ 8 (0.03 0.00 0.00 0.11 0.25 0.09 0.26 0.26)
- ☐ 9 (0.26 0.09 0.00 0.00 0.26 0.03 0.11 0.25)
- ☐ 10 Pas de distribution limite



Entraînement - Training

Noircissez complètement ci-dessous les 3 premières lettres de votre nom de famille et la première lettre de votre prénom. Par exemple, pour Jean Dupont, cochez J, D, U, P ; pour Henri Harley, cochez seulement H, A, R ; pour Bernard Ca, cochez seulement A, B, C.

Check entirely the 3 first letters of your lastname and the first letter of your firstname. For instance, for Jean Dupont, check J, D, U, P ; for Henri Harley, check only H, A, R ; for Bernard Ca, check only A, B, C.

A	B	C	D	E	F	G	H	I	J	K	L	M
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Then write your lastname and firstname below.

Nom et prénom :

.....

Les réponses aux questions sont à donner exclusivement sur cette feuille. Les réponses données sur les feuilles précédentes ne seront pas prises en compte. Pour cocher une case, il faut la **noircir complètement**. Vous pouvez effacer votre réponse à la gomme ou avec du blanc, attention à ne pas effacer la case à cocher. Si vous êtes dans l'impossibilité de corriger une erreur, cette page est dupliquée au verso ; vous pouvez alors barrer cette feuille ci et répondre au verso.

QUESTION 1 :

1	2	3	4	5	6	7	8
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QUESTION 2 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 3 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40																

QUESTION 4 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 5 :

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

QUESTION 6 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 7 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40																

QUESTION 8 :

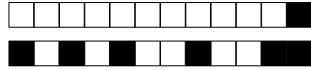
1	2	3	4	5	6	7	8	9	10
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QUESTION 9 :

1	2	3	4	5	6	7	8	9
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QUESTION 10 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 11 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40					

QUESTION 12 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 13 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 14 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 15 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40					

QUESTION 16 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 17 :

1	2	3	4	5	6	7	8	9
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QUESTION 18 :

1	2	3	4	5	6	7	8	9
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QUESTION 19 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40					

QUESTION 20 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 21 :

1	2	3	4	5	6	7	8	9
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QUESTION 22 :

1	2	3	4	5	6	7	8
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QUESTION 23 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40					

QUESTION 24 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 25 :

1	2	3	4	5	6	7	8	9
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QUESTION 26 :

1	2	3	4	5	6	7	8	9
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QUESTION 27 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40					

QUESTION 28 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 29 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 30 :

1	2	3	4	5	6	7	8	9
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QUESTION 31 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40					

QUESTION 32 :

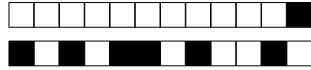
1	2	3	4	5	6	7	8	9	10
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QUESTION 33 :

1	2	3	4	5	6	7	8	9
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QUESTION 34 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 35 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15					
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
36	37	38	39	40															

QUESTION 36 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 37 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 38 :

1	2	3	4	5	6	7	8	9	10
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QUESTION 39 :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15					
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
36	37	38	39	40															

QUESTION 40 :

1	2	3	4	5	6	7	8	9	10
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