

The Maximum-2-Satisfiability problem

$$\begin{aligned}\varphi = & \underbrace{(x \vee \bar{z})}_{C_1:9} \wedge \underbrace{(\bar{x} \vee y)}_{C_2:10} \wedge \underbrace{(\bar{x} \vee \bar{y})}_{C_3:1} \wedge \underbrace{(x \vee \bar{y})}_{C_4:10} \\ & \wedge \underbrace{(x \vee t)}_{C_5:9} \wedge \underbrace{(\bar{x} \vee z)}_{C_6:6} \wedge \underbrace{(\bar{y} \vee t)}_{C_7:8} \wedge \underbrace{(\bar{x} \vee \bar{t})}_{C_8:6}\end{aligned}$$

What is the maximum weight of true clauses that can be obtained?

The Maximum-2-Satisfiability problem

$$\varphi = \underbrace{(x \vee \bar{z})}_{C_1:9} \wedge \underbrace{(\bar{x} \vee y)}_{C_2:10} \wedge \underbrace{(\bar{x} \vee \bar{y})}_{C_3:1} \wedge \underbrace{(x \vee \bar{y})}_{C_4:10}$$

$$\wedge \underbrace{(x \vee t)}_{C_5:9} \wedge \underbrace{(\bar{x} \vee z)}_{C_6:6} \wedge \underbrace{(\bar{y} \vee t)}_{C_7:8} \wedge \underbrace{(\bar{x} \vee \bar{t})}_{C_8:6}$$

16 possibilities

x	y	z	t	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Weight
T	T	T	T	T	T	F	T	T	T	T	F	52
T	T	T	F	T	T	F	T	T	T	F	T	50
T	T	F	T	T	T	F	T	T	F	T	F	46
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

The Maximum-2-Satisfiability problem

16 possibilities

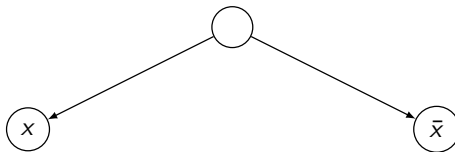
x	y	z	t	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	Weight
T	T	T	T	T	T	F	T	T	T	T	F	52
T	T	T	F	T	T	F	T	T	T	F	T	50
T	T	F	T	T	T	F	T	T	F	T	F	46
T	T	F	F	T	T	F	T	T	F	F	T	44
T	F	T	T	T	F	T	T	T	T	T	F	43
T	F	T	F	T	F	T	T	T	T	T	T	49
T	F	F	T	T	F	T	T	T	F	T	F	37
T	F	F	F	T	F	T	T	T	F	T	T	43
F	T	T	T	F	T	T	F	T	T	T	T	40
F	T	T	F	F	T	T	F	F	T	F	T	23
F	T	F	T	T	T	T	F	T	T	T	T	49
F	T	F	F	T	T	T	F	F	T	F	T	32
F	F	T	T	F	T	T	T	T	T	T	T	50
F	F	T	F	F	T	T	T	F	T	T	T	41
F	F	F	T	T	T	T	T	T	T	T	T	59
F	F	F	F	T	T	T	T	F	T	T	T	50

Optimal solution : $\bar{x}, \bar{y}, \bar{z}, t$

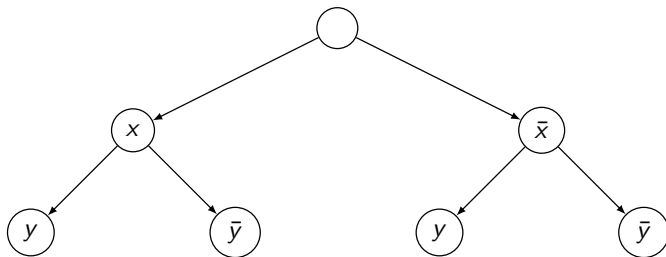
Tree representation : branching



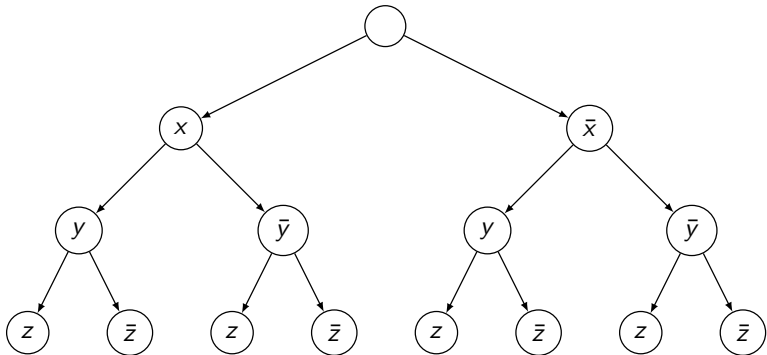
Tree representation : branching



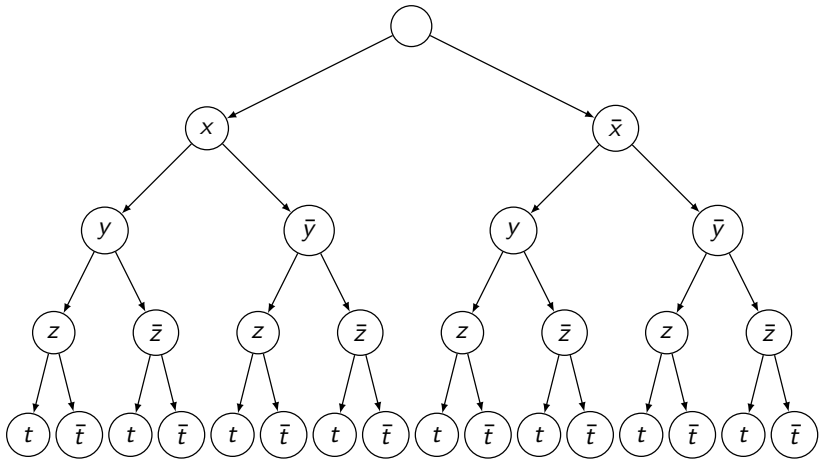
Tree representation : branching



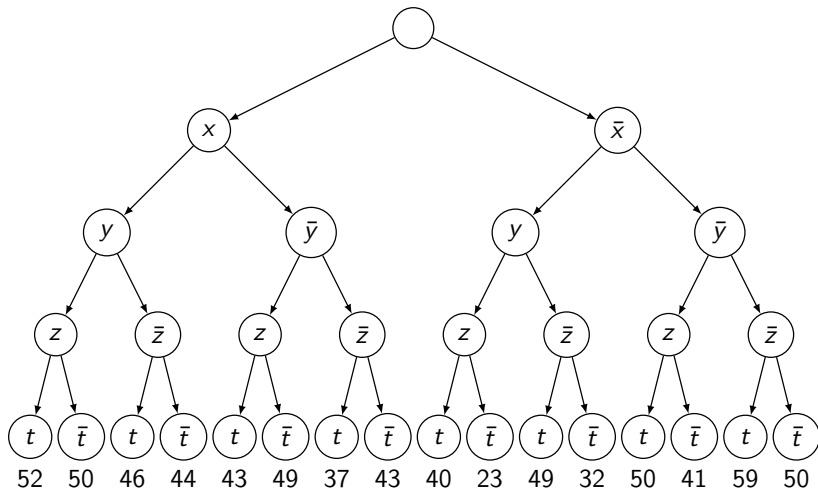
Tree representation : branching



Tree representation : branching



Tree representation : branching



Tree representation : bound

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If x and y were fixed to V and F , what maximum value can we obtain?

Tree representation : bound

$$\varphi = \underbrace{(x \vee \bar{z})}_{C_1:9} \wedge \underbrace{(\bar{x} \vee y)}_{C_2:10} \wedge \underbrace{(\bar{x} \vee \bar{y})}_{C_3:1} \wedge \underbrace{(x \vee \bar{y})}_{C_4:10}$$
$$\wedge \underbrace{(x \vee t)}_{C_5:9} \wedge \underbrace{(\bar{x} \vee z)}_{C_6:6} \wedge \underbrace{(\bar{y} \vee t)}_{C_7:8} \wedge \underbrace{(\bar{x} \vee \bar{t})}_{C_8:6}$$

If x and y were fixed to V and F , what maximum value can we obtain?

49 because C_2 can never be satisfied.

Tree representation : bound

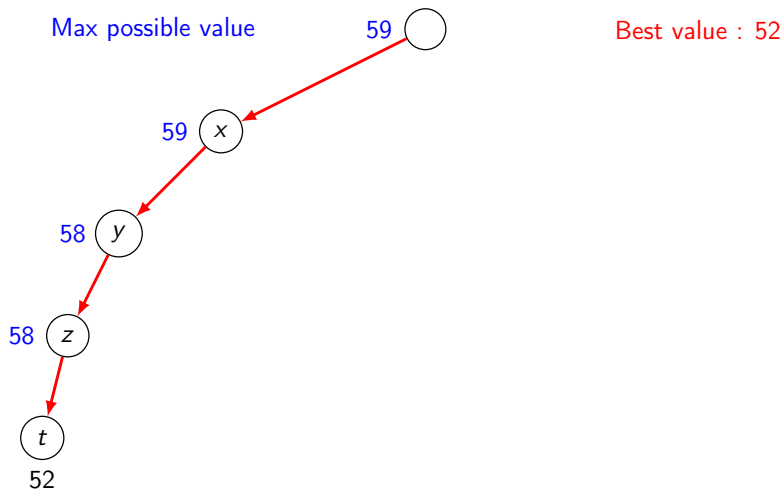
$$\varphi = \underbrace{(x \vee \bar{z})}_{C_1:9} \wedge \underbrace{(\bar{x} \vee y)}_{C_2:10} \wedge \underbrace{(\bar{x} \vee \bar{y})}_{C_3:1} \wedge \underbrace{(x \vee \bar{y})}_{C_4:10}$$

$$\wedge \underbrace{(x \vee t)}_{C_5:9} \wedge \underbrace{(\bar{x} \vee z)}_{C_6:6} \wedge \underbrace{(\bar{y} \vee t)}_{C_7:8} \wedge \underbrace{(\bar{x} \vee \bar{t})}_{C_8:6}$$

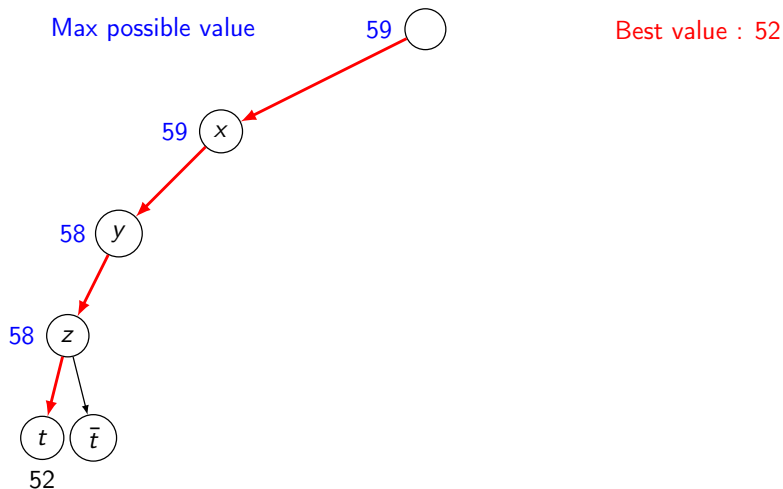
If x and y were fixed to V and F , what maximum value can we obtain?

It would be useless to explore that case if we already know a solution with value 49 or more.

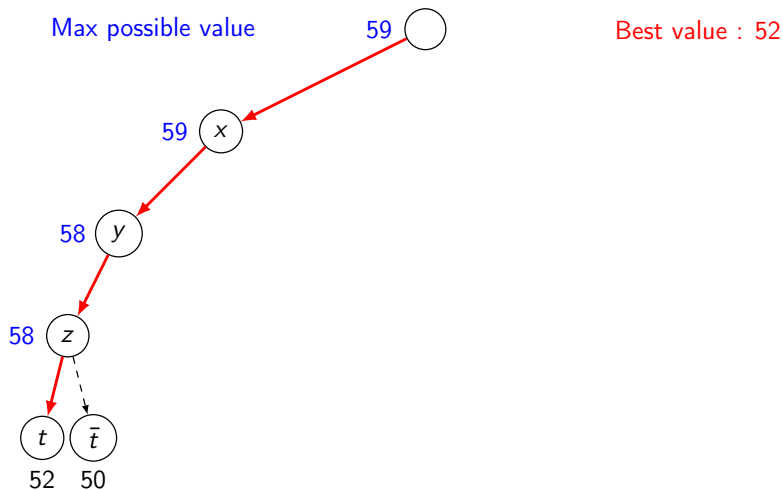
Tree representation : branch and bound



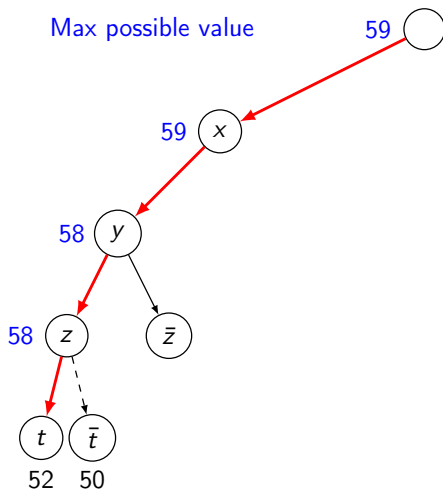
Tree representation : branch and bound



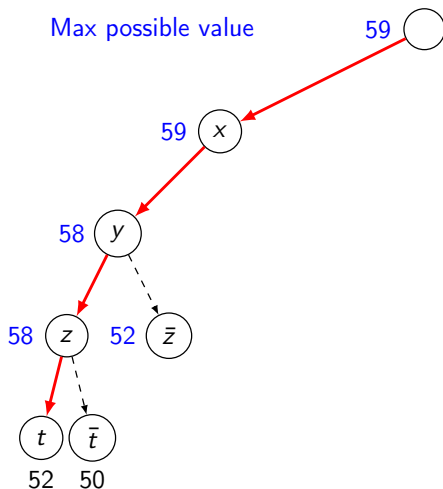
Tree representation : branch and bound



Tree representation : branch and bound

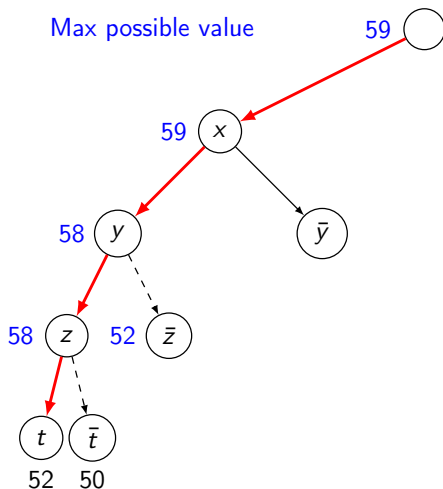


Tree representation : branch and bound

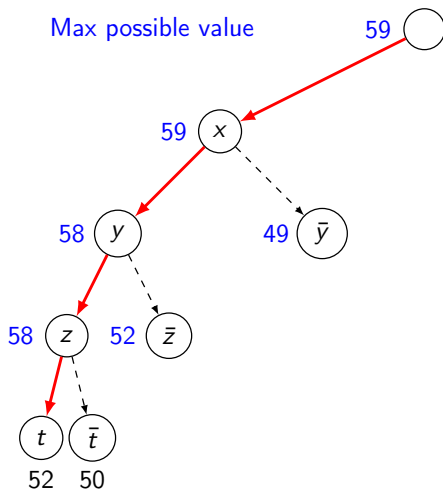


Best value : 52

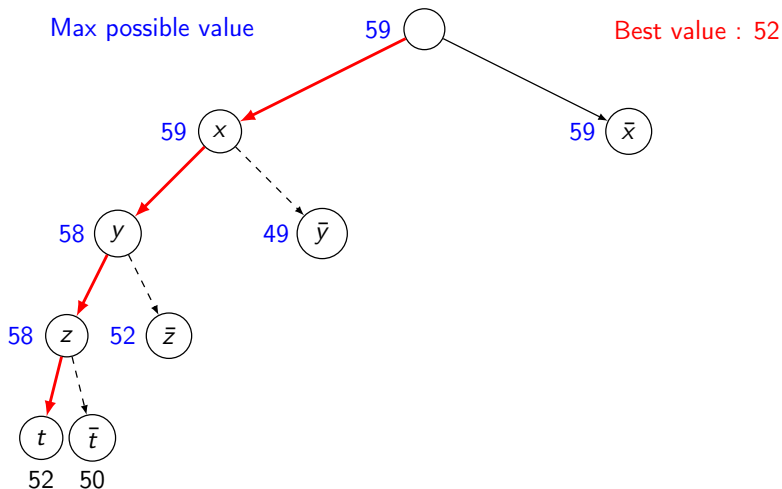
Tree representation : branch and bound



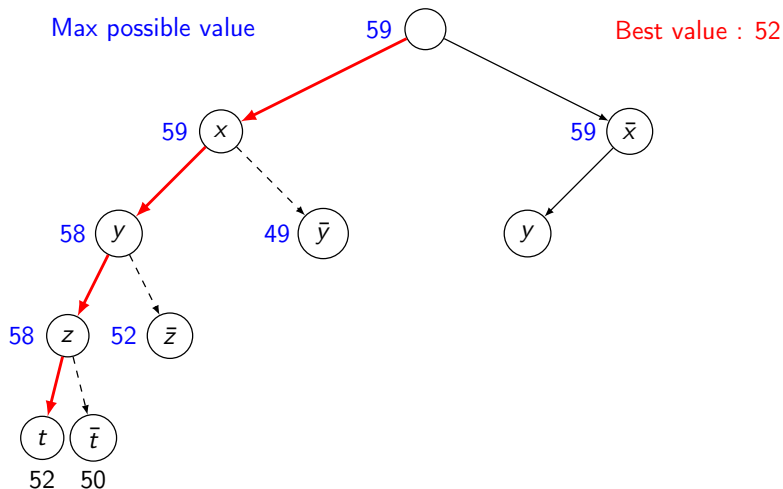
Tree representation : branch and bound



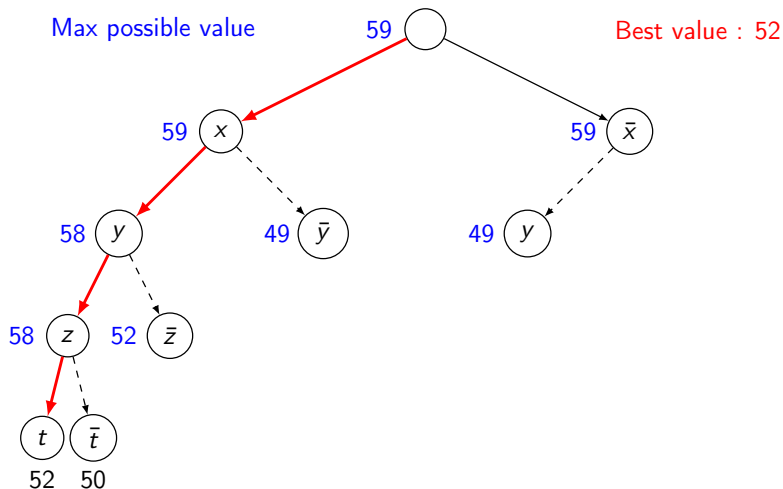
Tree representation : branch and bound



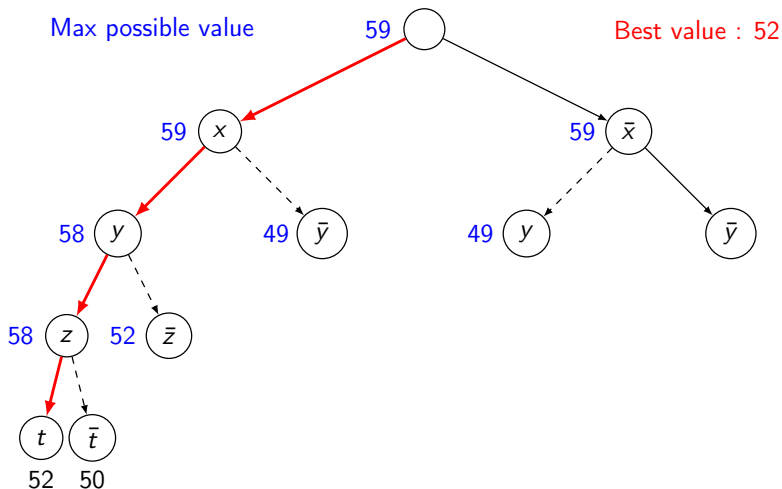
Tree representation : branch and bound



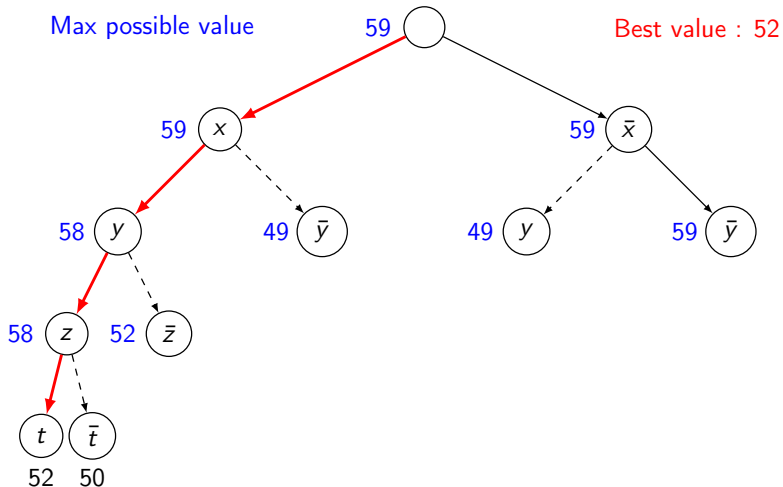
Tree representation : branch and bound



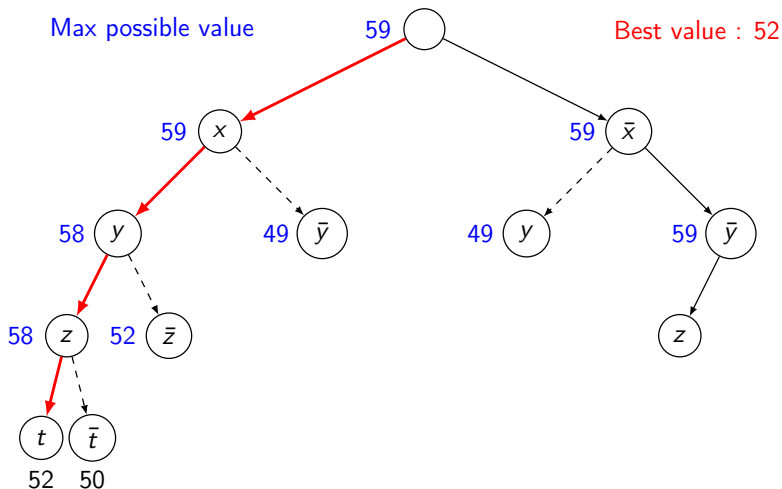
Tree representation : branch and bound



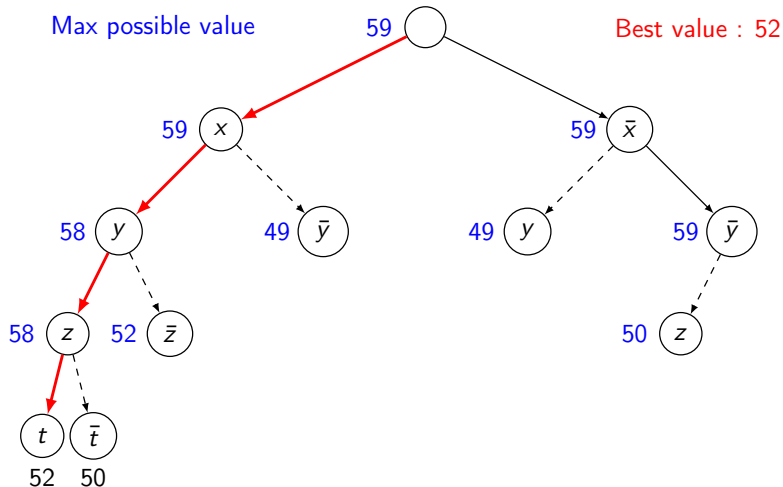
Tree representation : branch and bound



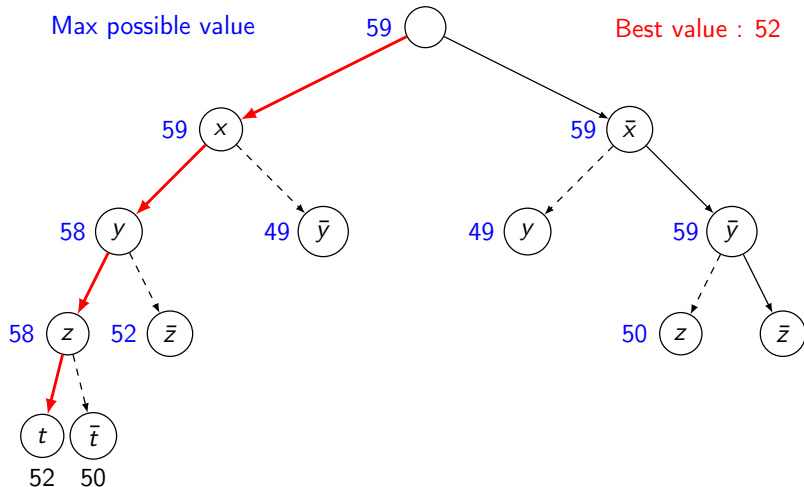
Tree representation : branch and bound



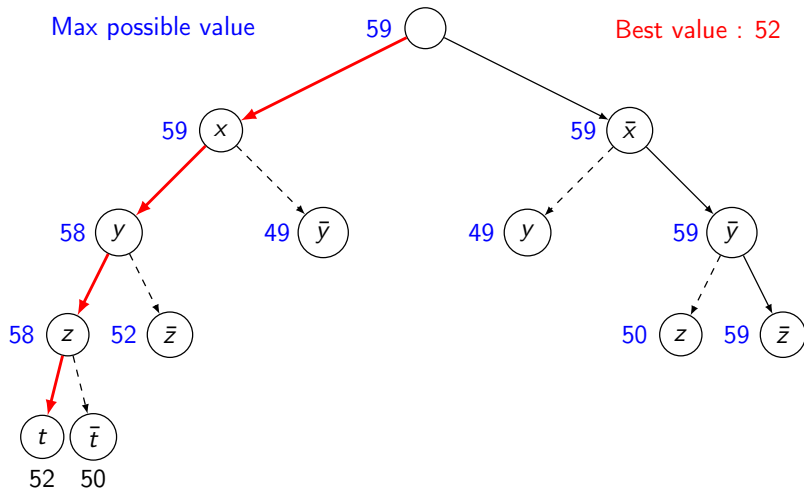
Tree representation : branch and bound



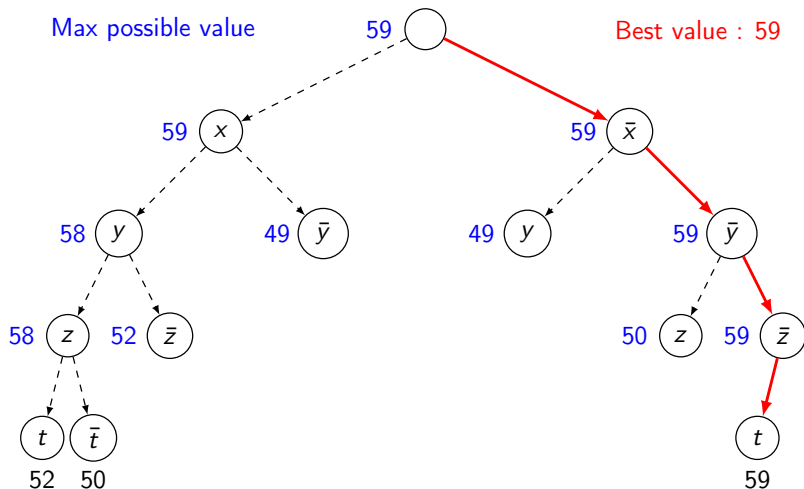
Tree representation : branch and bound



Tree representation : branch and bound



Tree representation : branch and bound



Tree representation : branch and bound

