Tutorial 3 : Production planning

Operations research, 3rd semester.

2024

Exercice 1 — *Flow Shop Problem*

Five jobs must be performed, on a machine M_1 and on a machine M_2, in any order. When the task of a job j on a machine is done, the other machine can start to work on the job j if and only if it is not currently working on another job. In that case, we must wait for the machine to be available before using it for j.

1. Give the optimal value and an optimal solution if the duration of each job on each machine is given on the following table.

Duration $t(j, M)$	j_1	j_2	j_3	j_4	j_5
M_1	2	4	1	8	2
M_2	1	3	2	10	1

2. Same question with the following durations.

Duration $t(j, M)$	j_1	j_2	j_3	j_4	j_5
M_1	3	4	2	5	3
M_2	4	3	2	1	5

Exercice 2 — *Flow Shop Problem (bis)*

Five jobs must be performed, firstly on a machine M_1 and then on a machine M_2, in that order. When the task of a job j on M_1 is done, the machine M_2 can start to work on the job j if and only if it is not currently working on another job. In that case, we must wait for M_2 to be available before using it for j. The duration of each job on each machine is given on the following table.

Duration $t(j, M)$	j_1	j_2	j_3	j_4	j_5
M_1	50	150	80	200	30
M_2	60	50	150	70	200

1. What is the total duration if we perform the jobs j_1, j_2, j_3, j_4, j_5 in that order.
2. Apply the Johnson algorithm in order to find an order that minimize the total duration of the execution.
3. What is the complexity of that algorithm ?
4. We assume the following fact as proved : a solution is optimal if for every couple of jobs i preceding j, $\min(t(i, M_1), t(j, M_2)) \leq \min(t(i, M_2), t(j, M_1))$. Show that a solution returned by the Johnson algorithm is optimal.

Exercice 3 — *Flow Shop Problem with 3 machines*

Same subject as the previous exercice but with 3 machines.

$t(j, M)$	j_1	j_2	j_3	j_4	j_5	j_6
M_1	60	40	80	70	100	50
M_2	40	20	10	30	20	30
M_3	40	60	70	100	50	80

1. We consider a fictive problem with only two machines M'_1 and M'_2 where, for each job j, $t(j, M'_1) = t(j, M_1) + t(j, M_2)$ and $t(j, M'_2) = t(j, M_2) + t(j, M_3)$. Apply the Johnson algorithm in order to find an optimal solution on that fictive problem.
2. What is the total duration of the execution of the fictive problem ?
3. What is the total duration of that same execution of the real problem ?
4. Reuse the algorithm on the following table

$t(j, M)$	j_1	j_2	j_3
M_1	10	20	40
M_2	70	80	20
M_3	20	00	35

5. Compare this solution with the following ordering $j_3 j_1 j_2$. Is this algorithm always optimal ?
6. The algorithm is optimal for the first example but not the second. What could explain this ?

Exercice 4 — *Project planning under resources constraints - heuristic.*

We consider the following project planning problem where the number of employees is 5.

task	duration	previous tasks	nb employees
A	6	-	3
B	3	-	2
C	6	-	1
D	2	B	1
E	4	B	3
F	3	D A	3
G	1	F E C	2

The PERT/Metra potential methods does not take into account any capacity constraint as the number of employees. The following algorithm, called the serial method or list algorithm, solve the problem. However it may not return an optimal solution : this is called a heuristic.

a) Define a priority order for the tasks satisfying the precedence constraint.
b) For each task t, in that order
   A) Consider the minimum time τ where the task t may be done (precedence and ressource constraint).
   B) Define the starting time of the task t as τ.

We are going to use this heuristic for our example.

1. By using the metra potential method, find an optimal ordering that ignore the resource constraint. Is this ordering satisfying that constraint ? Compute the late start of each task.
2. Apply the serial method. The priority of each task is its late start.
3. What is the GANTT diagram of that planning ?