Tutorial 4 : Branch and bound

Operations research, 3rd semester.

2024

Exercice 1 — *Constrainted selection*

The following table gives you the costs and utilities of Counter Strike weapons and accessories. The costs are in dollars and the utilities in points. You have 4200$. We want to maximize the utility of the equipment and satisfy the budget constraint. In order to simplify, we add the following constraint : in each column, you have to choose one (and only one) object.

	A : Pistol	B : Rifle	C : Grenade	D : Protection
1	Bereta 500$ 10pt	AK-47 2700$ 60 pt	Explosive 300$ 40pt	Kevlar 650$ 70pt
2	Desert Eagle 650$ 50pt	FAMAS 2250$ 40pt	Flash-Bang 200$ 20pt	Kevlar-Helmet 1000$ 80pt

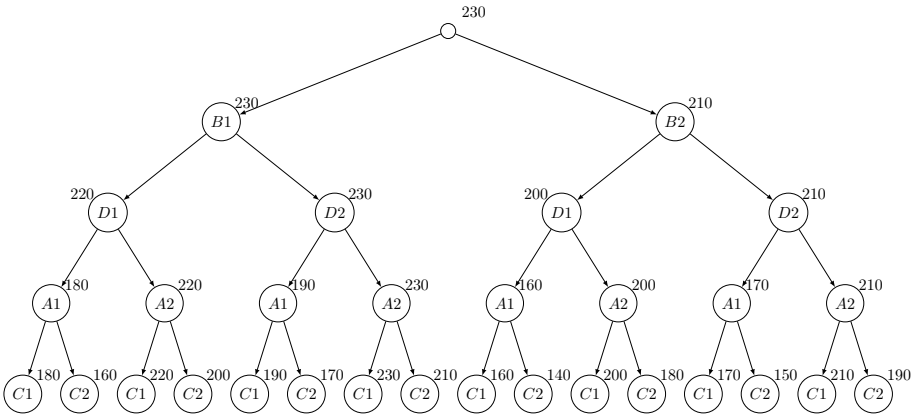
We want to solve the problem with a branch and bound method. At each iteration, we make a choice for a category. Each node of the following tree gives you the column and the line of the corresponding choice. For example, B1D2A1C1 corresponds to AK-47, Kevlar-Helmet, Bereta, Explosive grenade. The subtree of B2D2 is the set of solutions containing the FAMAS and the Kevlar-Helmet. We wrote an upper bound near each node : the maximum utility that can be reached considering the choices that were already made.

The formula of that bound is

— for each category in which a choice is already made, we add the utility of the chosen object ;

— for each other category, we add the maximum utility among all the objects of that category.

For example, for the root, no choice is made. The maximum utility we can achieve is 50+60+40+80 with the Desert Eagle, the AK-47, the Explosive grenades and the Kevlar-Helmet. For B2D2 we choose the FAMAS and the Kevlar-Helmet, of utility 40+80, and then, we add 50, the utility of the Desert Eagle, and 40, the utility of the Explosive grenades, (for the best pistol and the best grenade, according to the table). We then have 210.



1. How many nodes of the tree would explore an algorithm that does not take into account the upper bounds ? (It does not explore a node if the price is already larger than 4200$).
2. If you use the algorithm of the course, which use the depth first search strategy (the left child first, then the right child), in which order would the nodes be explored and how many nodes would you explore ?

3. If you use the depth first search strategy with the right child first, in which order would the nodes be explored and how many nodes would you explore ?
4. If you use the best first strategy (the node with the highest upper bound first), in which order would the nodes be explored and how many nodes would you explore ? Could you explain what happens ? Does it always happen when we use this strategy ?
5. If you use the worst first strategy (the node with the lowest upper bound first), in which order would the nodes be explored and how many nodes would you explore ?

Exercice 2 — *Project planning with ressources*

We consider the following project planning problem where the number of employees is 5. The same problem was already given in a previous tutorial. We want to know how much time we need to finish the projet while satisfying the comstraint with the number of employees.

task	duration	previous tasks	nb employees
A	6	-	3
B	3	-	2
C	6	-	1
D	2	B	1
E	4	B	3
F	3	D A	3
G	1	F E C	2

1. Draw the potential-metra graph of that project. If we do not consider the resources constraint, what is the minimum duration t of the project ? Why t is a lower bound of the optimal solution of the problem where we satisfy the resources constraint ?
2. What is the first time where the capacity constraint is violated ? Let T be the jobs that violate the constraint. Show that, in an optimal solution, there are at least two tasks t and t' of T such that t is before t'.
3. In order to branch the problem, we can do that way : for each couple of jobs (t, t') of T, we define a new sub-problem where t must be before t' (by adding a precedence constraint in the table). We then build $|T| \cdot (|T| - 1)$ sub-problems. Using that branching algorithm and the lower bound of question 1, could you find an optimal solution of the example of the exercice ?

Exercice 3 — *Late genetic*

Two students in the field of genetic are finishing a project in order to get their master diploma. They did experiments on ten mouses in which the color of the fur changes. Because they started the experiments the day before the deadline, they need to analyse the data as fast as possible so that they can finish the report on time.

Consequently, they decided to divide the analysis into two equal parts. However, it is hard to conclude anything from only half of the experiments. Indeed, each mouse had a more or less long contact with the other mouses, so that the way the color of the fur changed on one mouse can impact the color of the fur of the other mouses : the data are correlated.

How does those students must part the mouses so that the total impact of the mouses of one group to the mouses on the other group is minimized ? Model this problem by a graph problem and solve it using branch and bound. Do you know a better algorithm ?

The following table gives, for each couple of mouses, the contact duration in minutes between those two mouses. A line means that there were no significant contact.

	A	B	C	D	E	F	G	H
A	-	1	12	-	-	1	-	7
B	1	-	8	-	-	-	-	-
C	12	8	-	2	-	-	-	-
D	-	-	2	-	6	-	5	-
E	-	-	-	6	-	4	-	10
F	1	-	-	-	4	-	5	-
G	-	-	-	5	-	5	-	3
H	7	-	-	-	10	-	3	-